## 2.5 The Binomial Theorem

Evaluate.

1. $\binom{5}{2}$

$$= \frac{5!}{3!2!} = \frac{5 \cdot 4}{2} = 10$$

2. $\binom{7}{5} = 21$

3. $\binom{8}{2} = 28$

4. $\binom{100}{2} = 4950$

Expand the binomial.

5. $(x+2)^4$

$$= x^4 + 4x^3 \cdot 2 + 6x^2 \cdot 2^2 + 4x \cdot 2^3 + 2^4 = x^4 + 8x^3 + 24x^2 + 32x + 16$$

6. $(a-2)^5$

$$x^5 - 5x^4 \cdot 2 + 10x^3 \cdot 2^2 - 10x^2 \cdot 2^3 + 5x \cdot 2^4 - 2^5 = x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

Write the first three terms of the binomial expansion.

7. $(x-2y)^7$

$$= \binom{7}{0}x^7 + \binom{7}{1}x^6(-2y) + \binom{7}{2}x^5(-2y)^2 + \dots$$

$$= x^7 + 7x^6(-2)y + 21x^5 4y^2 + \dots$$

$$= x^7 - 14x^6 y + 84x^5 y^2 + \dots$$

8. $(2x+3)^9$

$$= \binom{9}{0}(2x)^9 + \binom{9}{1}(2x)^8(3) + \binom{9}{2}(2x)^7(3)^2 + \ldots$$

$$= 2^9x^9 + 9(2^8)x^8(3) + 36(2^7)x^7(3)^2 + \ldots$$
$$= 512x^9 + 9(256)x^8(3) + 36 \cdot 128x^7 \cdot 9 + \ldots$$
$$= 512x^9 + 6912x^8 + 41472x^7 + \ldots$$

9. Find the coefficient of $x^3y^4$ in the expansion of $(x-2y)^7$.

$$\binom{7}{3}x^3(-2y)^4 = 35x^3(-2)^4y^4 = 35 \cdot 16x^3y^4 = 560x^3y^4$$

Answer: 560

10. Find the coefficient of $x^6$ in the expansion of $(x-2)^9$.

$$\binom{9}{6}x^6(-2)^3 = 84x^6(-8) = -672x^6$$