

4.4 Exponential and Logarithmic Equations

Find all of the real solutions.

1. $\log(3x + 1) = 2$
2. $\log(x + 1) - \log(x) = 3$
3. $\log_2(x + 2) + \log_2(x - 2) = 5$
4. $\ln(x) + \ln(x + 2) = \ln 8$
5. $\ln x - \ln(x + 1) = \ln(x + 3) - \ln(x + 5)$
6. $2^{x-1} = 7$
7. $9 = e^{-3x}$
8. $7 = 2e^{3t}$
9. $5 = 7e^{2t}$
10. $2^x = 3^{x-1}$

11. Exponential Decay is given by the function

$$A(t) = A_0e^{-kt}$$

where A is the amount at time t , t is time in years, A_0 is the amount at time $t = 0$, and k is a constant that depends only on the material that is decaying exponentially.

The half-life, h , of a material is the time that it takes for half of a given amount of material to decay. Find a formula that gives the constant k in terms of the half-life h .

12. Carbon-14 decays exponentially with a half-life of $h = 5730$ years. Write the constant k in terms of the half-life. How long does it take 2.4 g of carbon-14 to be reduced to 1.3 g of carbon-14?

13. *Dating a Bone:* A piece of bone from an organism is found to contain 10% of the carbon-14 that it contained when it was living. How long ago was the organism alive?
14. *Radioactive Waste:* If 25 g of radioactive waste reduces to 20 g in 8000 years, then what is the half-life, h , for the radioactive element?
15. Interest compounded continuously is given by the formula

$$A = Pe^{rt}.$$

A principal balance P is invested at a constant yearly rate r compounded continuously. If it takes 5 years for the principal to double, then what is the yearly interest rate r ?

16. The population P of a city grows exponentially according to the function

$$P(t) = P_0e^{kt}$$

where $P_0 = 30,000$ is the population at the starting time of January 1, 2000, t is the number of years after that starting time, and k is a constant. Find the constant k if it takes 11 years for the population to double. What will be the population at the beginning of 2017?