4.2 Logarithmic Functions

1. Write the exponential equation as a logarithmic equation.

   (a) $2^x = 3$ SOLUTION $\log_2 3 = x$

   (b) $10^{2x} = 7$ SOLUTION $\log 7 = 2x$

2. Write the logarithmic equation as an exponential equation. Do not solve for x.

   (a) $4 = \log 1000$ SOLUTION $10^4 = 1000$

   (b) $\log_2 x = 5$ SOLUTION $2^5 = x$

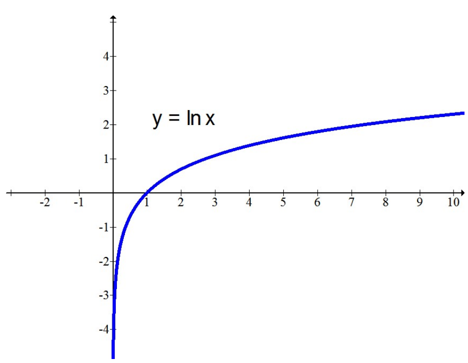
   (c) $\ln 3x = 7$ SOLUTION $e^7 = 3x$

   (d) $\log 9x = -3$ SOLUTION $10^{-3} = 9x$

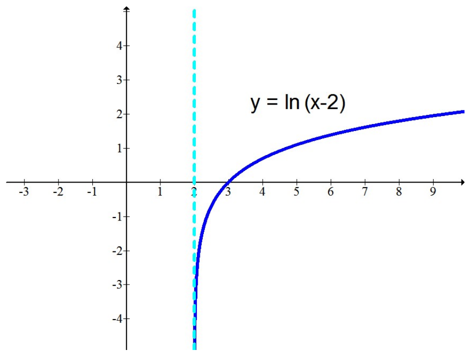
   (e) $-3 = \log_2 \left(\frac{1}{8}\right)$ SOLUTION $2^{-3} = \frac{1}{8}$

3. Sketch the graph. Label the intercepts and the asymptotes.

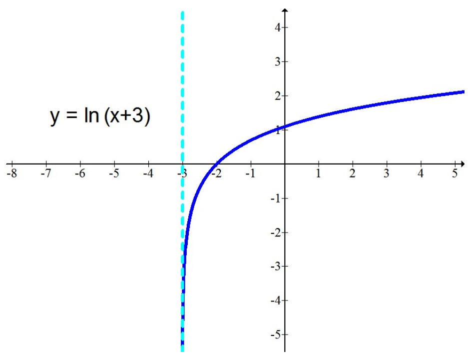
   (a) $y = \ln x$



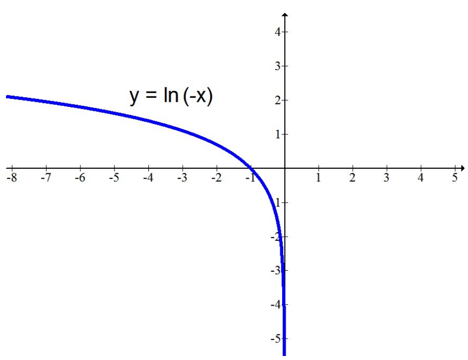
   (b) $y = \ln(x - 2)$



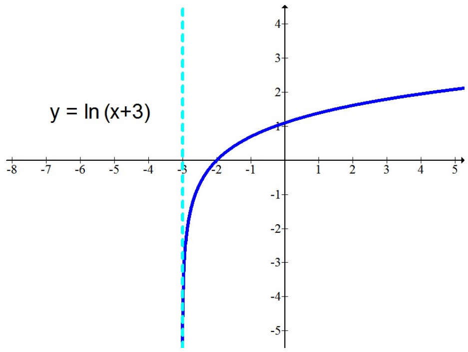
(c) $y = \ln(x + 3)$



(d) $y = \ln(-x)$



(e) $y = \ln(x + 3)$



4. Use a calculator to find a decimal approximation. Round your answer to the third decimal place.

   (a) $\ln 2 \approx 0.693$

   (b) $\log 2 \approx 0.301$

   (c) $\ln(1/2) \approx -0.693$

   (d) $\log(1/2) \approx -0.301$

   (e) $\ln e = 1$

   (f) $\log 10 = 1$

   (g) $\ln 1 = 0$

   (h) $\log 1 = 0$

   (i) $\log(10^4) = 4$

   (j) $\log(10^{-3}) = -3$

   (k) $\log(1,000,000) = 6$

5. The amount A of an investment of principal P compounded continuously at an annual rate r is given by the formula $A(t) = Pe^{rt}$. If \$1000 is invested at a rate of 6% compounded continuously, then how long will it take for the investment to grow to \$1700?

   Solution

$$A = 1700, \quad P, \quad r = .06$$

$$1700 = 1000e^{.06t}$$
$$\frac{1700}{1000} = e^{.06t}$$
$$1.7 = e^{.06t}$$
$$\ln 1.7 = .06t$$
$$t = \frac{\ln 1.7}{.06} \approx 8.84 \text{ years}$$

6. The balance amount A of an investment of principal P compounded continuously at an annual rate r is given by the formula $A(t) = Pe^{rt}$. If $r = 10\%$, then how long will it take for an investment of $1,000$ to double to $2,000$?

   SOLUTION

$$A = 2000, \quad P = 1000, \quad r = .1$$

$$2000 = 1000e^{.06t}$$
$$\frac{2000}{1000} = e^{.1t}$$
$$2 = e^{.1t}$$
$$\ln 2 = .1t$$
$$t = \frac{\ln 2}{.1} \approx 6.9 \text{ years}$$