4.2 Logarithmic Functions

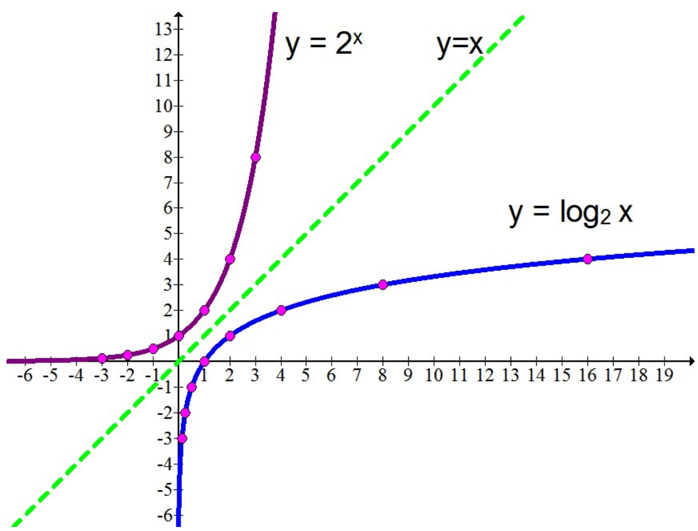
The function $y = 2^x$ is one-to-one. We know this because it passes the horizontal line test. Therefore the function $y = 2^x$ has an inverse function. We call the inverse function the logarithmic function with base 2, and we denote this inverse function as

$$y = \log_2 x.$$

Recall that we find the inverse function is by interchanging the x and y coordinates of the function. Therefore the inverse of a general exponential function $y = a^x$, $a > 1$, is the function $y = \log_a x$ such that

$$y = \log_a x \quad \text{is equivalent to } a^y = x$$

Recall that we find the graph of the inverse function by reflecting about the line $y = x$. The graph of $y = \log_2 x$ is shown below.



For any base $a > 1$, the graph of a general log function $y = \log_a x$ will look similar. Let's list some properties that are common to all log functions $y = \log ax$ where $a > 1$.

- Domain: All real numbers x such that $x > 0$
- Range: All real numbers

- Increasing
- One-to-one
- Invertible, with inverse $y = a^x$
- x-intercept $(0, 1)$
- Vertical Asymptote: The y-axis.

Going from Logarithms to Exponentials and Vice Versa

Example Write the logarithmic equation as an equation with exponents.

1. $3 = \log_2 8$

   SOLUTION $2^3 = 8$
2. $7 = \log_3 a$

   SOLUTION $3^7 = a$
3. $b = \log_5 c$
   SOLUTION $5^b = c$

Example Write the exponential equation as an equation with logarithms.

1. $2^x = 9$

   SOLUTION $\log_2 9 = x$
2. $3^{4t} = 17$

   SOLUTION $\log_3 17 = 4t$
3. $5^{3t} = c$
   SOLUTION $3t = \log_5 c$

The Common Log and the Natural Log

The logarithmic function, base 10, is used so often that it is called the **common logarithmic function**.

$$y = \log x \quad \text{means } y = \log_{10} x$$

The logarithmic function, base e, where $e \approx 2.7183$, is very convenient to use in calculus. It is called the natural logarithmic function.

$$y = \ln x \quad \text{means } y = \log_e x$$

The graphs of $y = \log x$ and $y = \ln x$ look very similar to the graph of $y = 2^x$. This is because the graphs of $y = 10^x$, $y = e^x$, and $y = 2^x$ all look very similar.

Sketching Transformations of Logarithmic Functions

Example Sketch the graph of the given function. Label x-intercepts and vertical asymptotes.

1. $y = \ln x$
2. $y = \ln(x - 2)$
3. $y = \log(x + 1)$
4. $y = \ln(-x)$
5. Bonus: $y = \ln |x|$

Solving Log Equations

Example Solve the given equation.

1. $\log x = 2$
2. $\log_2 8 = x$
3. $\ln x = 4$
4. $20 = 2e^{3t}$
5. $15 = 3e^{-4t}$