

§ 2.6 #1 Prove by induction.

$$1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2}$$

Proof

• Base Case
 $n=1$

$$1 = \frac{1(1+1)}{2}$$

$$1 = 1 \quad \checkmark$$

• Assume true for $n=k$, shows true for $n=k+1$

$$n=k \quad 1 + 2 + 3 + 4 + \dots + k = \frac{k(k+1)}{2} + (k+1)$$

$$1 + 2 + 3 + \dots + k + (k+1) = \frac{k(k+1)}{2} + \frac{(k+1) \cdot 2}{2}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$n=k+1 \quad 1 + 2 + 3 + \dots + k + (k+1) = \frac{(k+1)(k+2)}{2} = \frac{(k+1)((k+1)+1)}{2} \quad \square$$

§2.6 #2

Prove by induction

$$2 + 4 + 6 + \dots + 2n = n(n+1)$$

Proof

• Base Case

$$n=1$$

$$2 = 1(1+1)$$

$$2 = 2 \quad \checkmark$$

• Assume true for $n=k$, show true for $n=k+1$

$$n=k \quad 2 + 4 + 6 + \dots + 2k = k(k+1)$$

$$+ 2(k+1)$$

$$+ 2(k+1)$$

$$2 + 4 + 6 + \dots + 2k + 2(k+1) = k(k+1) + 2(k+1)$$

$$= k^2 + k + 2k + 2$$

$$= k^2 + 3k + 2$$

$$= (k+1)(k+2)$$

$$n=k+1$$

$$2 + 4 + 6 + \dots + 2k + 2(k+1) = (k+1)((k+1)+1)$$

□

§2.6 #3

SOLUTION

⊙ Prove by induction

$$3 + 7 + 11 + \dots + (4n-1) = n(2n+1)$$

Pf

• Base case
 $n=1$

$$3 \stackrel{?}{=} 1(2 \cdot 1 + 1)$$

$$3 = 3 \quad \checkmark$$

• Assume true for $n=k$. Show true for $n=k+1$

$$\begin{aligned} \underline{n=k} \quad 3 + 7 + 11 + \dots + (4k-1) &= k(2k+1) \\ &\quad + (4(k+1)-1) \end{aligned}$$

$$\begin{aligned} 3 + 7 + 11 + \dots + (4k-1) + (4(k+1)-1) &= k(2k+1) + (4(k+1)-1) \\ &= 2k^2 + k + 4k + 4 - 1 \\ &= 2k^2 + 5k + 3 \end{aligned}$$

$$\begin{aligned} &= (k+1)(2k+3) \\ &= (k+1)(2k+2+1) \end{aligned}$$

$$\underline{n=k+1} \quad 3 + 7 + 11 + \dots + (4k-1) + (4(k+1)-1) = (k+1)(2(k+1)+1)$$

□