

3.6 Rational Functions

1. Add the two given rational functions to get a rational function over a common denominator.

$$(a) R(x) = \frac{x-4}{x+3} \text{ and } S(x) = \frac{2x+1}{x-5}$$

$$\begin{aligned} & \frac{(x-4)}{(x+3)} + \frac{(2x+1)}{(x-5)} \\ &= \frac{(x-4)(-5+x)}{(x+3)(-5+x)} + \frac{(2x+1)(3+x)}{(x-5)(3+x)} \text{ make common denominator} \\ &= \frac{(20-9x+x^2)}{(-15-2x+x^2)} + \frac{(3+7x+2x^2)}{(-15-2x+x^2)} \text{ multiply} \\ &= \frac{(20-9x+x^2+3+7x+2x^2)}{(-15-2x+x^2)} \text{ collect terms} \\ &= \frac{(23-2x+3x^2)}{(-15-2x+x^2)} \text{ add} \\ &= \frac{3x^2-2x+23}{(x-5)(x+3)} \end{aligned}$$

$$\begin{aligned} (b) R(x) &= \frac{3x-1}{2x+2} \text{ and } S(x) = \frac{x-4}{3x+2} \\ &= \frac{11x^2-3x-10}{2(x+1)(3x+2)} \end{aligned}$$

$$\begin{aligned} (c) R(x) &= \frac{x^2-1}{x+5} \text{ and } S(x) = \frac{3x^3+2}{x+1} \\ &= \frac{3x^4+16x^3+x^2+x+9}{(x+1)(x+5)} \end{aligned}$$

$$\begin{aligned} (d) R(x) &= \frac{x^2-x+1}{3x-4} \text{ and } S(x) = \frac{x+3}{x^2+2} \\ &= \frac{x^4-x^3+6x^2+3x-10}{(3x-4)(x^2+2)} \end{aligned}$$

$$(e) R(x) = \frac{x^3 - 2x}{x + 4} \text{ and } S(x) = \frac{x^4 - 2}{3x + 1}$$

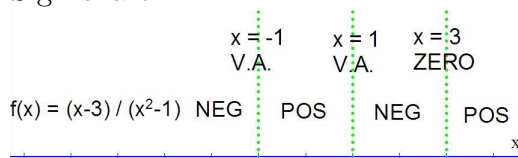
$$= \frac{x^5 + 7x^4 + x^3 - 6x^2 - 4x - 8}{(x + 4)(3x + 1)}$$

2. For the given rational function,

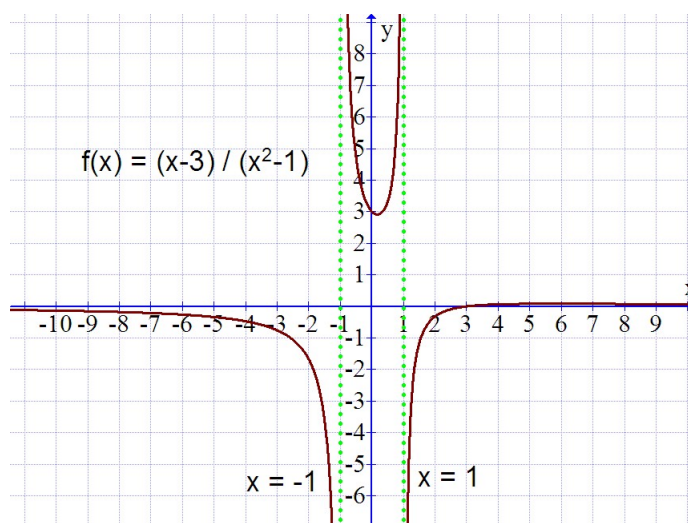
- i) Find the zeros.
- ii) Find the vertical asymptotes.
- iii) Find the horizontal or oblique asymptotes.
- iv) Make a sign chart.
- v) Sketch the graph.

$$(a) y = \frac{x - 3}{x^2 - 1}$$

- i) Zero $x = 3$
- ii) Vertical asymptotes the lines $x = 1, x = -1$
- iii) Horizontal asymptotes the x -axis.
- iv) Sign chart.

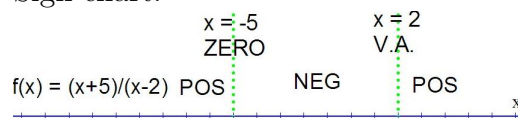


- v) Graph.

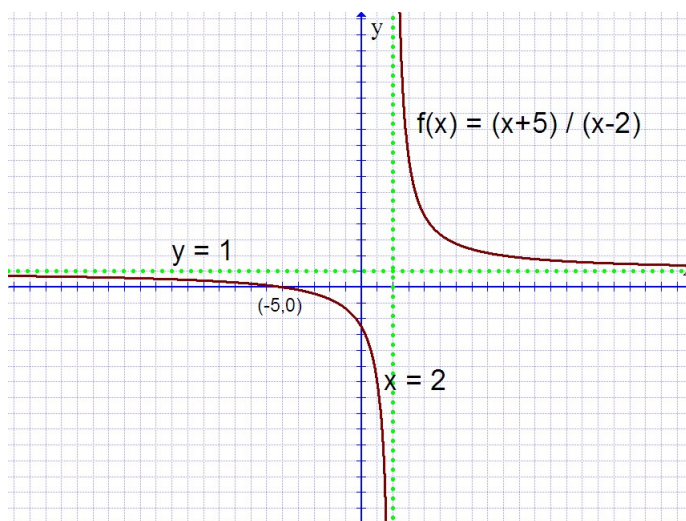


(b) $y = \frac{x+5}{x-2}$

- i) Zero $x = -5$
- ii) Vertical asymptotes the line $x = 2$
- iii) Horizontal asymptotes the line $y = 1$
- iv) Sign chart.

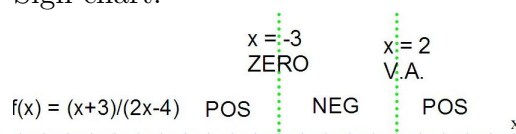


- v) Graph.

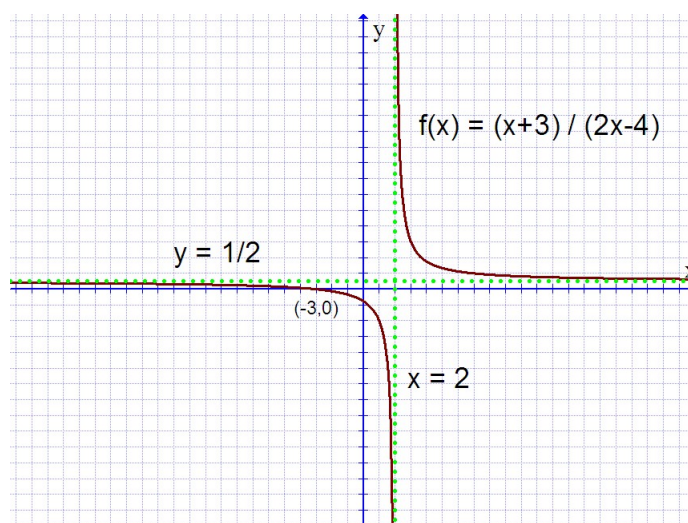


(c) $y = \frac{x + 3}{2x - 4}$

- i) Zeros. $x = 3$
- ii) Vertical asymptotes the line $x = 2$
- iii) Horizontal asymptotes the line $y = 1/2$
- iv) Sign chart.

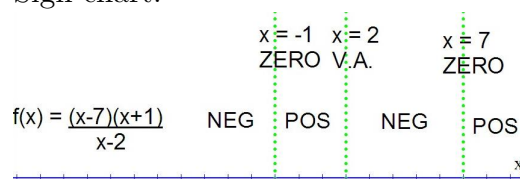


- v) Graph.

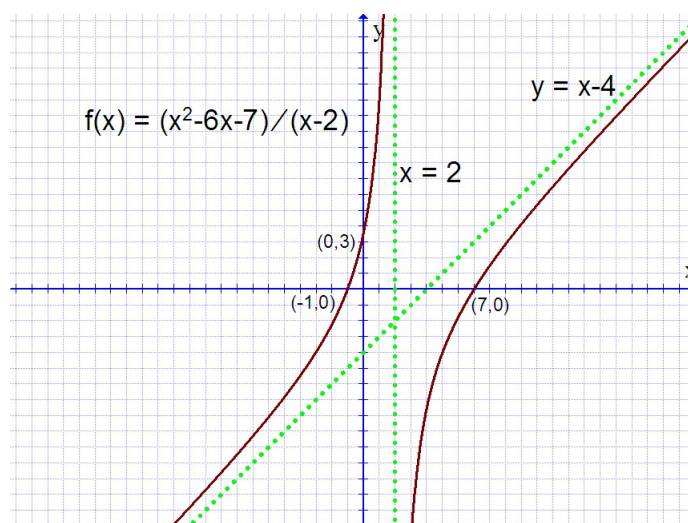


(d) $y = \frac{x^2 - 6x - 7}{x - 2}$

- i) Zeros $x = 7, x = -1$
- ii) Vertical asymptotes the line $x = 2$
- iii) Oblique asymptotes the line $y = x - 4$
- iv) Sign chart.



- v) Graph.



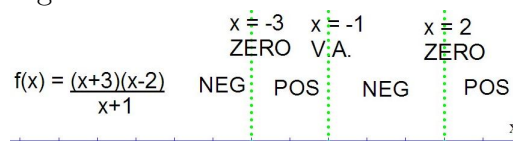
(e) $y = \frac{x^2 + x - 6}{x + 1}$

i) Zero $x = -3, x = 2$

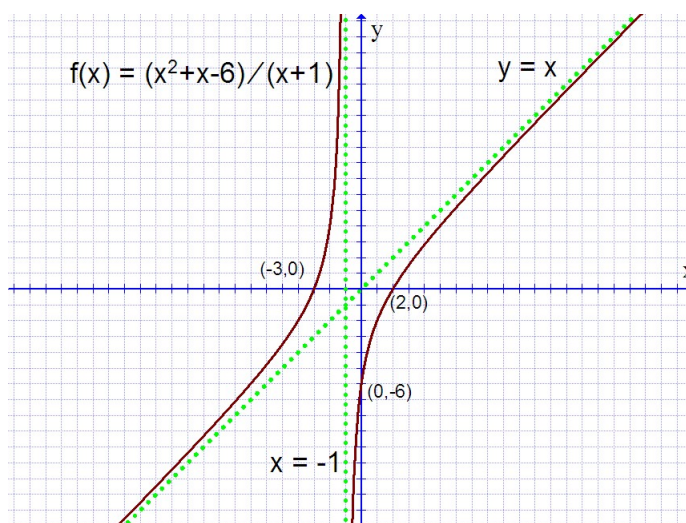
ii) Vertical asymptotes the line $x = -1$

iii) Oblique asymptotes. The line $y = x$

iv) Sign chart.

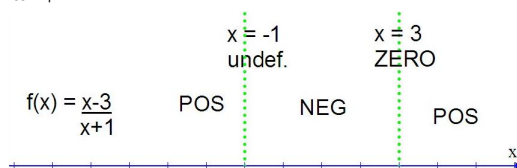


v) Graph.



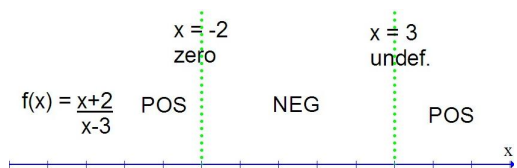
3. Solve the rational inequality.

(a) $\frac{x - 3}{x + 1} > 0$



Answer: $(-\infty, -1) \cup (3, \infty)$

(b) $\frac{x + 2}{x - 3} \leq 0$



Answer: $[-2, 3)$

(c) $\frac{1}{x + 3} > 0$

