

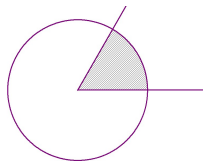
## 5.1 Angles and Their Measurements

1. Let  $R_{PQ}$  be a ray as drawn, horizontally.



Draw a second ray  $R_{PM}$  such that the angle  $\angle QPM$  has:

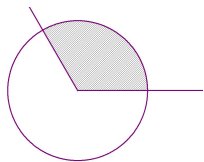
- (a)  $60^\circ$



$$\theta = 60^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{\pi}{3}, \quad r = 3$$

$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2} \left( (3)^2 \frac{\pi}{3} \right) = \frac{3\pi}{2}$$

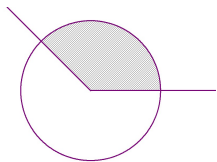
- (b)  $120^\circ$



$$\theta = 120^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{2\pi}{3}, \quad r = 3$$

$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2} \left( (3)^2 \frac{2\pi}{3} \right) = 3\pi$$

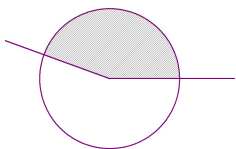
- (c)  $135^\circ$



$$\theta = 135^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{3\pi}{4}, \quad r = 3$$

$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2} \left( (3)^2 \frac{3\pi}{4} \right) = \frac{27\pi}{8}$$

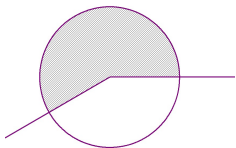
- (d)  $160^\circ$



$$\theta = 160^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{8\pi}{9}, \quad r = 3$$

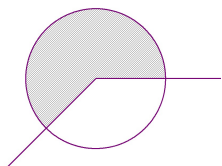
$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2} \left( (3)^2 \frac{8\pi}{9} \right) = 4\pi$$

- (e)  $210^\circ$

(f)  $225^\circ$ 

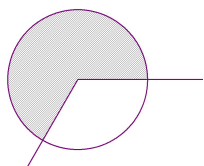
$$\theta = 210^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{7\pi}{6}, \quad r = 3$$

$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2}\left((3)^2\frac{7\pi}{6}\right) = \frac{21\pi}{4}$$

(g)  $240^\circ$ 

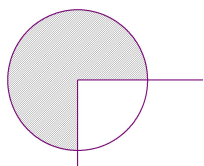
$$\theta = 225^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{5\pi}{4}, \quad r = 3$$

$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2}\left((3)^2\frac{5\pi}{4}\right) = \frac{45\pi}{8}$$

(h)  $270^\circ$ 

$$\theta = 240^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{4\pi}{3}, \quad r = 3$$

$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2}\left((3)^2\frac{4\pi}{3}\right) = 6\pi$$



$$\theta = 270^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{3\pi}{2}, \quad r = 3$$

$$\text{Area of Sector: } A = \frac{1}{2}r^2\theta = \frac{1}{2}\left((3)^2\frac{3\pi}{2}\right) = \frac{27\pi}{4}$$

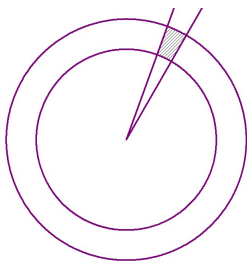
2. Let  $D$  be a disc centered at  $P$  with radius 3. In each one of the above cases find the area of the sector in the disc cut out by the two rays.

Area of sector,  $\theta$  in radians:  $A = \frac{r^2\theta}{2}$ . The solutions are shown above.

3. What is the area of the region lying between two circles having the same center, of radii 3 and 4, and bounded by angles of:

(a)  $60^\circ$  and  $70^\circ$

$$\theta = 70^\circ - 60^\circ = 10^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{\pi}{18}$$



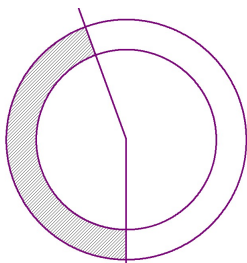
$$\text{Big Sector: } r = 4, \quad A_2 = \frac{1}{2}r^2\theta = \frac{1}{2} \left( (4)^2 \frac{\pi}{18} \right)$$

$$\text{Small Sector: } r = 3, \quad A_1 = \frac{1}{2}r^2\theta = \frac{1}{2} \left( (3)^2 \frac{\pi}{18} \right)$$

$$\text{Area of Region} = A_2 - A_1 = \frac{1}{2} \left( (4)^2 \frac{\pi}{18} \right) - \frac{1}{2} \left( (3)^2 \frac{\pi}{18} \right) = \frac{1}{2} (16 - 9) \frac{\pi}{18} = \frac{7\pi}{36}$$

$$\text{FORMULA: Area of Region} = \frac{1}{2}(r_2^2 - r_1^2)\theta, \quad \theta = \theta_2 - \theta_1 \text{ in radians}$$

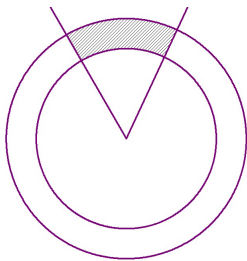
(b)  $110^\circ$  and  $270^\circ$



$$\theta = 270^\circ - 110^\circ = 160^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{8\pi}{9}$$

$$\text{Area of Region} = \frac{1}{2}(r_2^2 - r_1^2)\theta = \frac{1}{2}(4^2 - 3^2) \frac{8\pi}{9} = \frac{28\pi}{9}$$

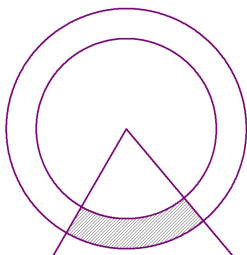
(c)  $65^\circ$  and  $120^\circ$



$$\theta = 120^\circ - 65^\circ = 55^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{11\pi}{36}$$

$$\text{Area of Region} = \frac{1}{2}(r_2^2 - r_1^2)\theta = \frac{1}{2}(4^2 - 3^2) \frac{11\pi}{36} = \frac{77\pi}{72}$$

(d)  $240^\circ$  and  $310^\circ$



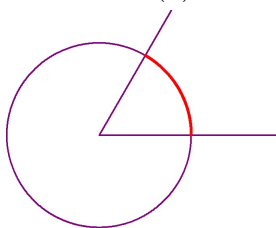
$$\theta = 310^\circ - 240^\circ = 70^\circ \cdot \left(\frac{\pi}{180}\right) = \frac{7\pi}{18}$$

$$\text{Area of Region} = \frac{1}{2}(r_2^2 - r_1^2)\theta = \frac{1}{2}(4^2 - 3^2)\frac{7\pi}{18} = \frac{49\pi}{36}$$

4. What is the length of the arc  $s$  of a circle of the given radius  $r$  intercepted by the given angle of measure  $\theta$  radians.

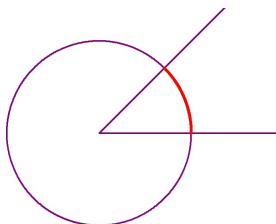
Area of Arc:  $s = r\theta$ ,  $\theta$  in radians

- (a)  $r = 3$  and  $\theta = \pi/3$  radians



$$\text{Area of Arcs} = r\theta = 3\frac{\pi}{3} = \pi$$

- (b)  $r = 5$  and  $\theta = \pi/4$  radians



$$\text{Area of Arcs} = r\theta = \frac{5\pi}{4}$$

5. Give the following values of angle in radians, as a fractional multiple of  $\pi$ .

(a)  $15^\circ = 15^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{12}$

(b)  $75^\circ = 5\pi/12$

(c)  $105^\circ = 7\pi/12$

(d)  $120^\circ = 2\pi/3$

(e)  $135^\circ = 3\pi/4$

(f)  $150^\circ = 5\pi/6$

(g)  $165^\circ = 11\pi/12$

(h)  $20^\circ = \pi/9$

(i)  $40^\circ = 2\pi/9$

(j)  $140^\circ = 7\pi/9$

(k)  $310^\circ = 31\pi/18$

6. Find the measure in degrees (between  $0^\circ$  and  $360^\circ$ ) for the following angles given in radians.

(a)  $-\frac{\pi}{4} = \left(-\frac{\pi}{4}\right) \frac{180^\circ}{\pi} = -45^\circ$

(b)  $\frac{8\pi}{9} = 160^\circ$

(c)  $\frac{5\pi}{9} = 100^\circ$

(d)  $\frac{7\pi}{4} = 315^\circ$

(e)  $\frac{14\pi}{3} = 840^\circ$

(f)  $\frac{22\pi}{3} = 1320^\circ$

(g)  $-\frac{\pi}{3} = -60^\circ$