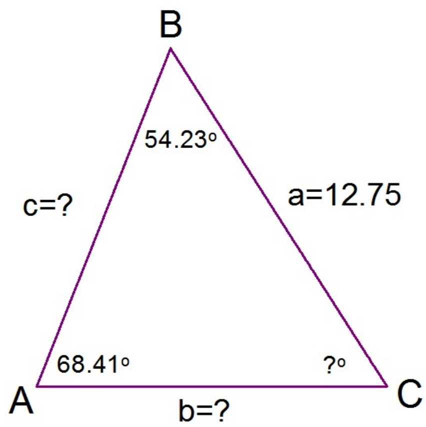
7.1 The Law of Sines

Determine the remaining sides and angles of each triangle ABC.

1. $A = 68.41°,\ B = 54.23°,\ a = 12.75$

Solution

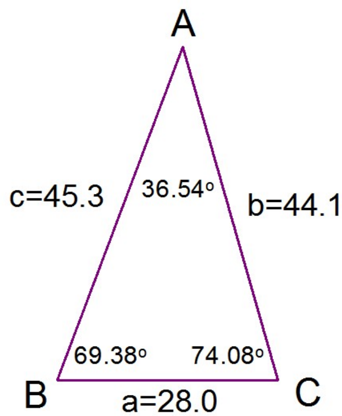


Find $\angle C$. $\angle C = 180° - 68.41° - 54.23° = 57.36°$.

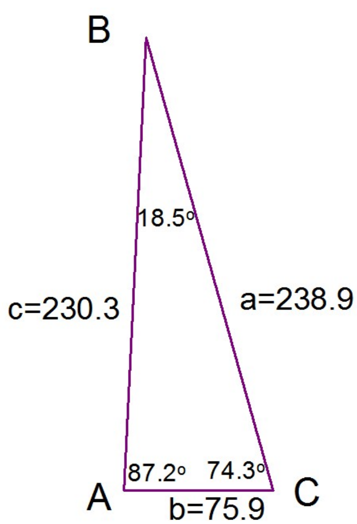
Find b.
$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} \\ b &= \sin B \cdot \frac{a}{\sin A} = \sin 54.23° \cdot \frac{12.75}{\sin 68.41°} \approx 11.13 \\ b &\approx 11.13 \end{aligned}$$

Find c.
$$\begin{aligned} \frac{a}{\sin A} &= \frac{c}{\sin C} \\ c &= \sin C \cdot \frac{a}{\sin A} = \sin 57.36° \cdot \frac{12.75}{\sin 68.41°} \approx 11.55 \\ c &\approx 11.55 \end{aligned}$$

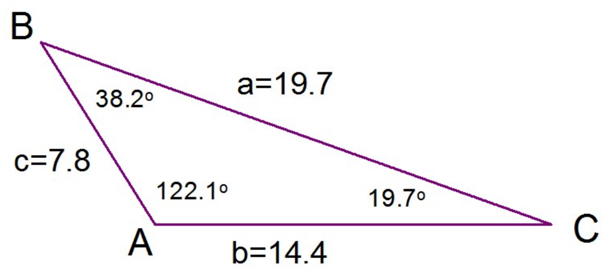
2. $C = 74.08°, \ B = 69.38°, \ c = 45.3$



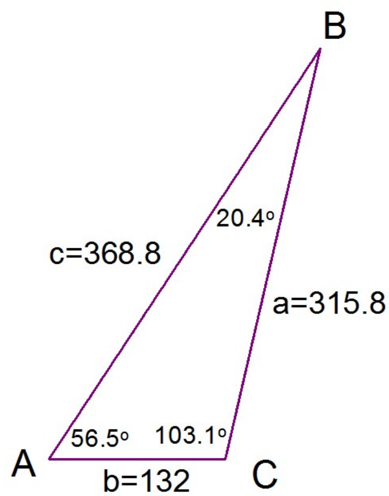
3. $A = 87.2°, \ b = 75.9, \ C = 74.3°$



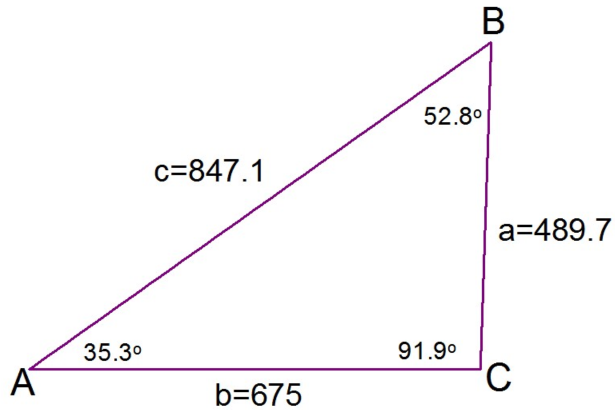
4. $B = 38.2°, \ a = 19.7, \ C = 19.7°$



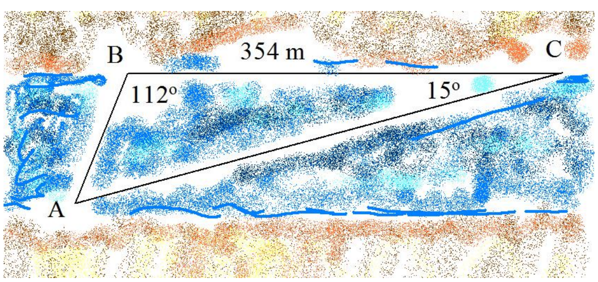
5. $B = 20.4°, \; C = 103.1°, \; b = 132$



6. $A = 35.3°, \; B = 52.8°, \; b = 675$

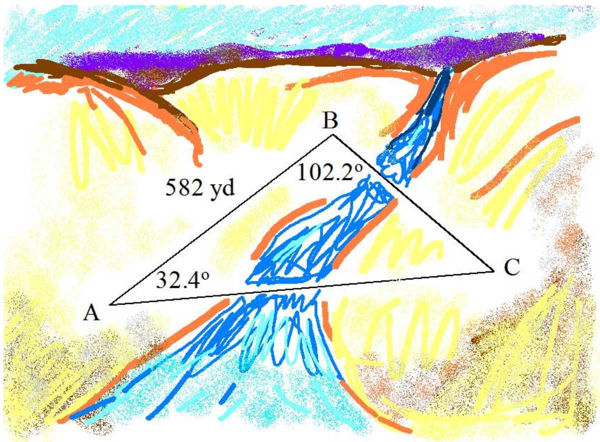


7. *Distance Across a River.* To find the distance AB across a river, a surveyor laid off a distance $BC = 354$ m on one side of the river. It is found that $B = 112°$ and $C = 15°$. Find AB.



SOLUTION $AB = 114.7$

8. *Distance Across a Canyon.* To determine the distance BC across a deep canyon, Rhonda lays off a distance $AB = 582$ yd. She then finds that $A = 32.4°$ and $B = 102.2°$. Find BC.



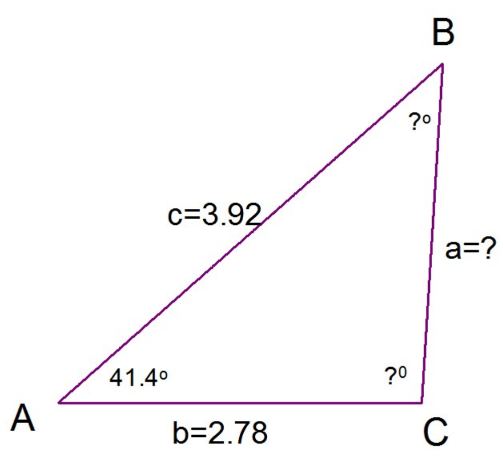
SOLUTION $BC = 438$ yd.

7.2 The Law of Cosines

Solve each triangles.

9. $A = 41.4°,\ b = 2.78,\ c = 3.92$

SOLUTION



Find a. $a^2 = b^2 + c^2 - 2bc \cos A$

$$
\begin{aligned}
a &= \sqrt{b^2 + c^2 - 2bc\cos A} \\
a &= \sqrt{(2.78)^2 + (3.92)^2 - 2(2.78)(3.92)\cos 41.4^\circ} \\
a &\approx 2.60
\end{aligned}
$$

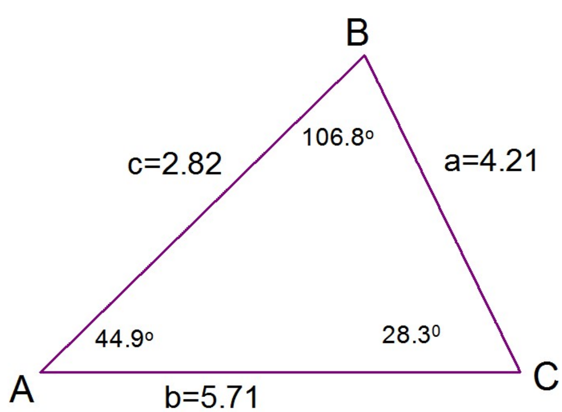
Note that $\angle C$ may be an obtuse angle (greater than 90°) because it is opposite the longest side. We do not want to use inverse sine to find an acute angle because the range of inverse sine is $(-90^\circ, 90^\circ)$. So let's use the Law of Sines, and inverse sine, to find $\angle B$.

<u>Find $\angle B$.</u>

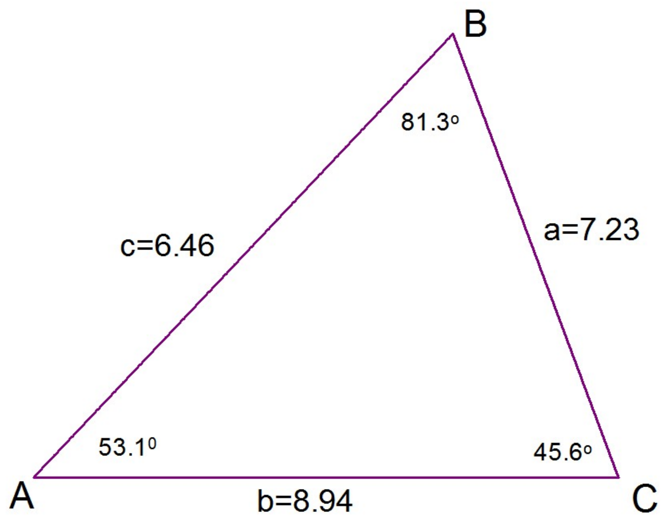
$$
\begin{aligned}
\frac{\sin A}{a} &= \frac{\sin B}{b} \\
\sin B &= b \cdot \frac{\sin A}{a} = 2.78 \cdot \frac{\sin 41.4^\circ}{2.60} \\
\angle B &= \sin^{-1}\left(2.78 \cdot \frac{\sin 41.4^\circ}{2.60}\right) \approx 45.1^\circ \\
\angle B &\approx 52.1^\circ
\end{aligned}
$$

<u>Find $\angle C$.</u> $\angle C = 180^\circ - 41.4^\circ - 52.1^\circ = 93.4^\circ$

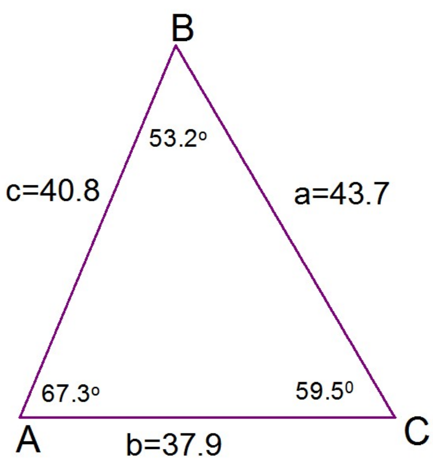
10. $C = 28.3^\circ,\ b = 5.71,\ a = 4.21$



11. $C = 45.6°,\ b = 8.94,\ a = 7.23$

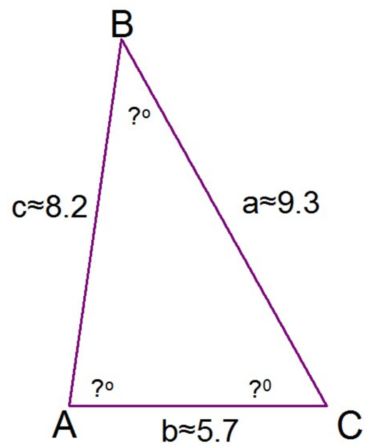


12. $A = 67.3°,\ b = 37.9,\ c = 40.8$



13. $a = 9.3,\ b = 5.7,\ c = 8.2$

SOLUTION



$$\begin{aligned}
\underline{\text{Find } \angle A.} \quad a^2 &= b^2 + c^2 - 2bc\cos A \\
\cos A &= \frac{a^2 - b^2 - c^2}{-2bc} \\
\angle A &= \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) \\
\angle A &= \cos^{-1}\left(\frac{(9.3)^2 - (5.7)^2 - (8.2)^2}{-2(5.7)(8.2)}\right) \\
\angle A &\approx 81.9^\circ
\end{aligned}$$

$$\begin{aligned}
\underline{\text{Find } \angle B.} \quad \frac{\sin A}{a} &= \frac{\sin B}{b} \\
\sin B &= b \cdot \frac{\sin A}{a} = 5.7 \cdot \frac{\sin 81.9^\circ}{9.3} \\
\angle B &= \sin^{-1}\left(5.7 \cdot \frac{\sin 81.9^\circ}{9.3}\right) \approx 37.4^\circ \\
\angle B &\approx 37.4^\circ
\end{aligned}$$

$$\underline{\text{Find } \angle C.} \quad \angle C = 180^\circ - 81.9^\circ - 37.4^\circ = 60.7^\circ$$

14. $a = 28,\ b = 47,\ c = 58$

SOLUTION $\angle A \approx 28.6^\circ,\ \angle B \approx 53.3^\circ,\ \angle C \approx 98.1^\circ$

15. $a = 42.9,\ b = 37.6,\ c = 62.7$

    SOLUTION $\angle A \approx 42.0,\ \angle B \approx 35.9^\circ,\ \angle C \approx 102.1^\circ$

16. $a = 189,\ b = 214,\ c = 325$

    SOLUTION $\angle A \approx 33.7^\circ,\ \angle B \approx 38.9^\circ,\ \angle C \approx 107.3^\circ$