

## 7.1 Oblique Triangles and the Law of Sines

A **right triangle** is a triangle that has a  $90^\circ$  angle. If a triangle is not a right triangle, then it is called an **oblique triangle**.

There are three sides and three angles in a triangle. This makes six pieces of information in total. When we **solve a triangle**, we are finding the length of all three sides and the measure of all three angles. To solve a triangle, we only need to know three pieces of information. Our goal in this section is to solve oblique triangles.

There are five possible cases: side-angle-angle (SAA), angle-side-angle (ASA), angle-side-side (ASS), side-angle-side (SAS), and side-side-side (SSS).

We can use the Law of Sines to Solve the SAA and the ASA case. In the case of ASS, we can use the Law of Sines, but there are several possible outcomes, and the process long. This is called the ambiguous case, and we are leaving it out of these notes. An explanation of the ambiguous case ASS can be found in most trigonometry textbooks. We use the Law of Cosines to solve the SAS and SSS cases.

### Law of Sines

In any triangle  $ABC$ , with sides  $a, b$ , and  $c$ ,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or equivalently

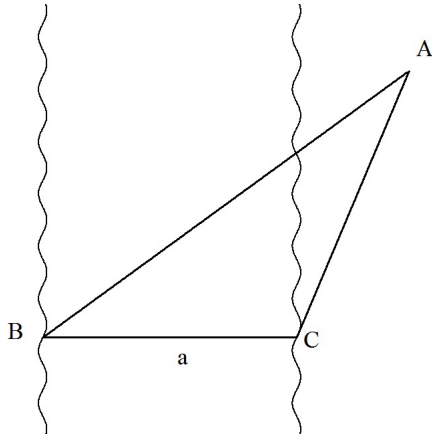
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### Solving SSA and ASA Triangles

**Example** SAA Case. Solve  $\triangle ABC$  if  $A = 32.0^\circ$ ,  $B = 81.8^\circ$ , and  $a = 42.9$  cm.

**Example** ASA Case. Jerry Keefe wishes to measure the distance across the

Big Muddy River. For  $\triangle ABC$ , he determines that  $C = 112.90^\circ$ ,  $A = 31.10^\circ$ , and  $c = 347.6$  ft. Find the distance  $a$  across the river.



## 7.2 The Law of Cosines

### The Law of Cosines

In any triangle  $ABC$ , with sides  $a, b$  and  $c$ ,

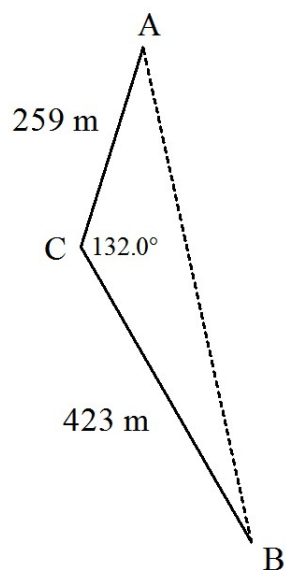
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

We can use the Law of Cosines to solve triangles for the SAS and SSS cases.

**Example** SAS Case. A surveyor wishes to find the distance between two inaccessible points  $A$  and  $B$  on opposite sides of a lake. While standing at point  $C$ , she finds that  $AC = 259$  m,  $BC = 423$  m, and angle  $ACB$  measures  $132.0^\circ$ . Find the distance  $AB$ .



**Example** SAS Case. Solve triangle  $ABC$  if  $A = 42.3^\circ$ ,  $b = 12.9$  m, and  $c = 15.4$  m.

**Example** SSS Case. Solve triangle  $ABC$  if  $a = 9.47$  ft,  $b = 15.9$  ft, and  $c = 21.1$  ft.

**Example** Designing a Roof Truss (SSS). Find angle  $B$  for the truss.

