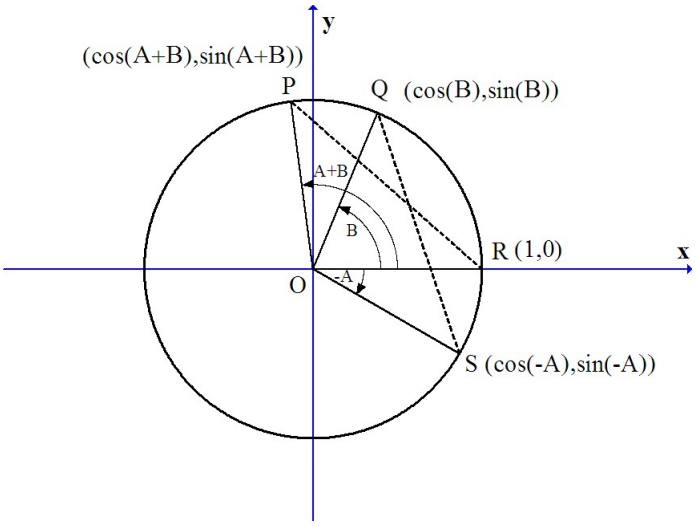
6.3 Sum and Differences Identities

Sum Identity for Cosine

Cosine of a Sum or Difference

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$



$$|PR| = |QS|$$

Example Finding Exact Cosine Function Values. Find the *exact* value of each expression.

1. $\cos 15°$

2. $\cos \frac{5\pi}{12}$

3. $\cos 87° \cos 93° - \sin 87° \sin 93°$

Example Write $\cos\left(\frac{\pi}{3} - \theta\right)$ as a trigonometric function of θ alone.

Cofunction Identities

Co-Function Identities
For any acute angle θ,

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \quad \sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \quad \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right) \quad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right) \quad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

Example Using Cofunction Identities. Find an angle θ that satisfies the following.

$$\cot\theta = \tan\left(\frac{\pi}{8}\right)$$

Sum and Difference Identities for Sine

Sine of a Sum or Difference

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

Sum and Difference Identities for Tangent

Tangent of a Sum or Difference

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Example Suppose that α and β are angles in standard position, with $\sin\alpha = 4/5$, angle α in quadrant II, and $\cos\beta = -5/13$, angle β in quadrant III. Find the following.

1. $\sin(\alpha + \beta)$
2. $\cos(\alpha + \beta)$
3. The quadrant in which the angle $\alpha + \beta$ lies.