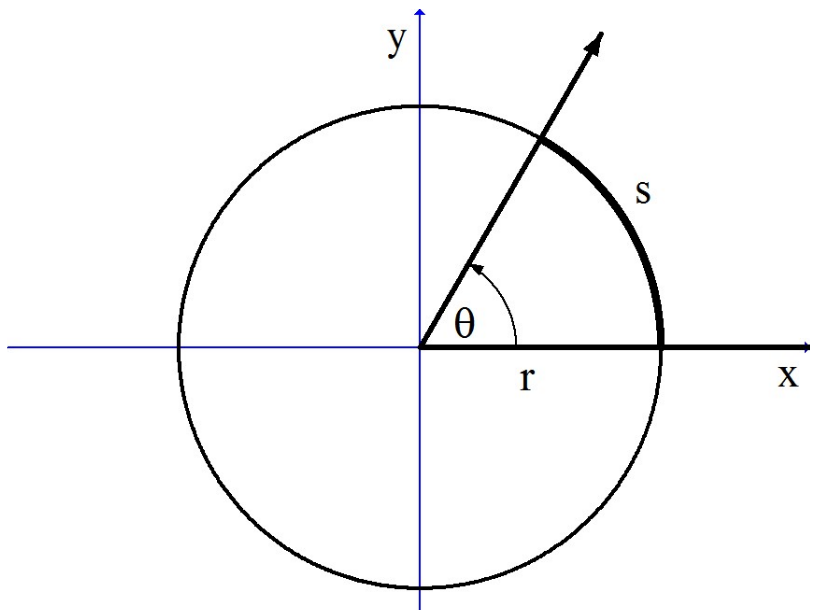
5.4 Tangent, Cotangent, Secant, and Cosecant

Definition Six Trigonometric Functions

If (x, y) is the point of intersection of the terminal side of an angle θ in standard position on the **unit circle**, then we define the six trigonometric functions.

$$\begin{array}{lcl} \text{cosine } \theta & = & x \\ \text{sine } \theta & = & y \\ \text{tangent } \theta & = & y/x \end{array} \qquad \begin{array}{lcl} \text{secant } \theta & = & 1/x \\ \text{cosecant } \theta & = & 1/y \\ \text{cotangent } \theta & = & x/y \end{array}$$



The following identities follow immediately from the definition of the six trigonometric functions.

Identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Reciprocal Identities:

$$\cot\theta = \frac{1}{\tan\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\csc\theta = \frac{1}{\sin\theta}$$

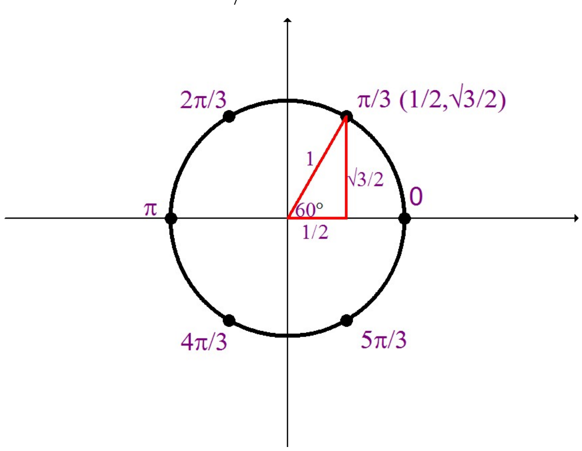
If we know the values of sine and cosine, then we can find the values of the remaining trig functions using the identities.

Example Evaluate the following expressions without using a calculator.

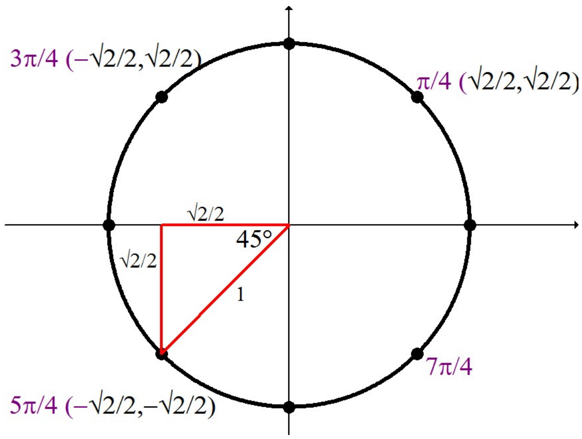
1. $\tan\left(\frac{\pi}{3}\right)$
2. $\sec\left(\frac{5\pi}{4}\right)$
3. $\csc\left(\frac{4\pi}{3}\right)$
4. $\cot\left(\frac{7\pi}{4}\right)$

SOLUTION

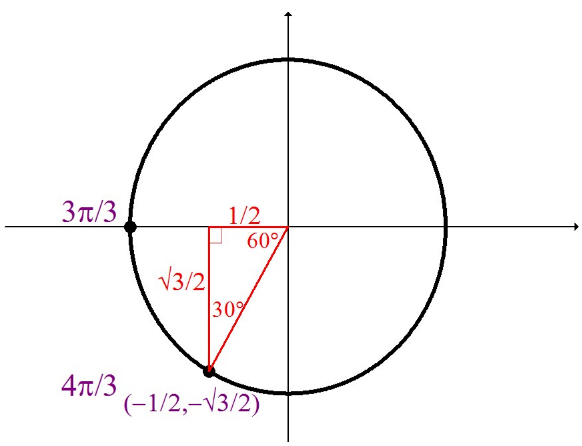
1. $\tan\frac{\pi}{3} = y/x = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$



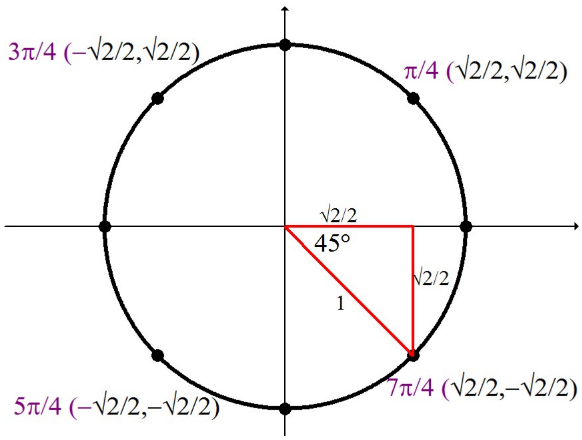
2. $\sec\left(\frac{5\pi}{4}\right) == \frac{1}{x} = \frac{1}{-\sqrt{2}/2} = -\frac{2}{\sqrt{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$



3. $\csc\left(\frac{4\pi}{3}\right) = \frac{1}{y} = \frac{1}{-\sqrt{3}/2} = -\frac{2}{\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$



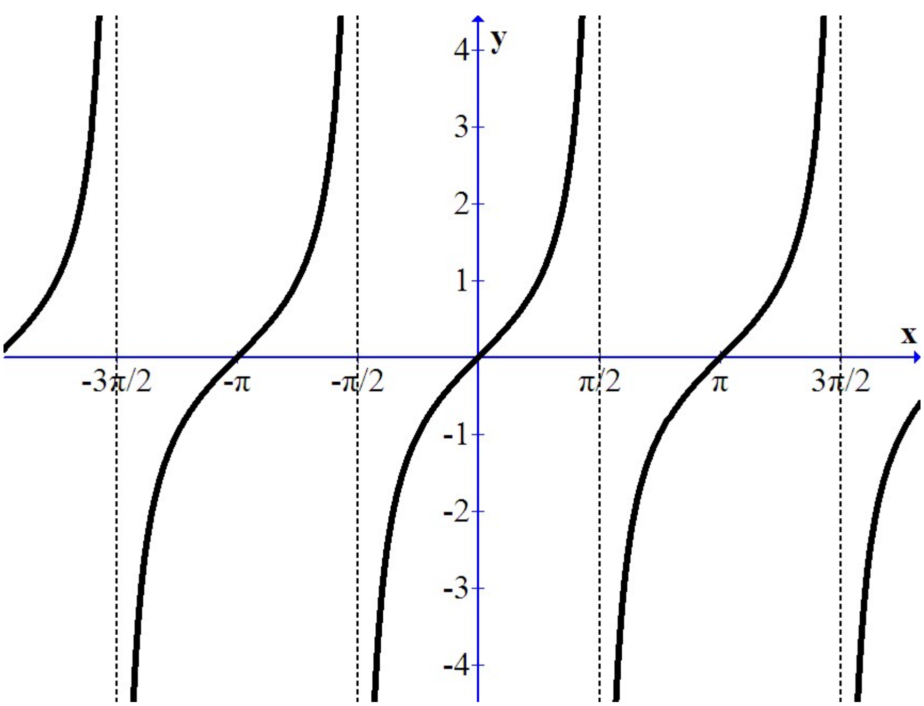
4. $\cot\left(\frac{7\pi}{4}\right) = \frac{x}{y} = \frac{\sqrt{2}/2}{-\sqrt{2}/2} = -1$



Graphs of Tangent, Cotangent, Secant, and Cosecant

The Tangent Graph

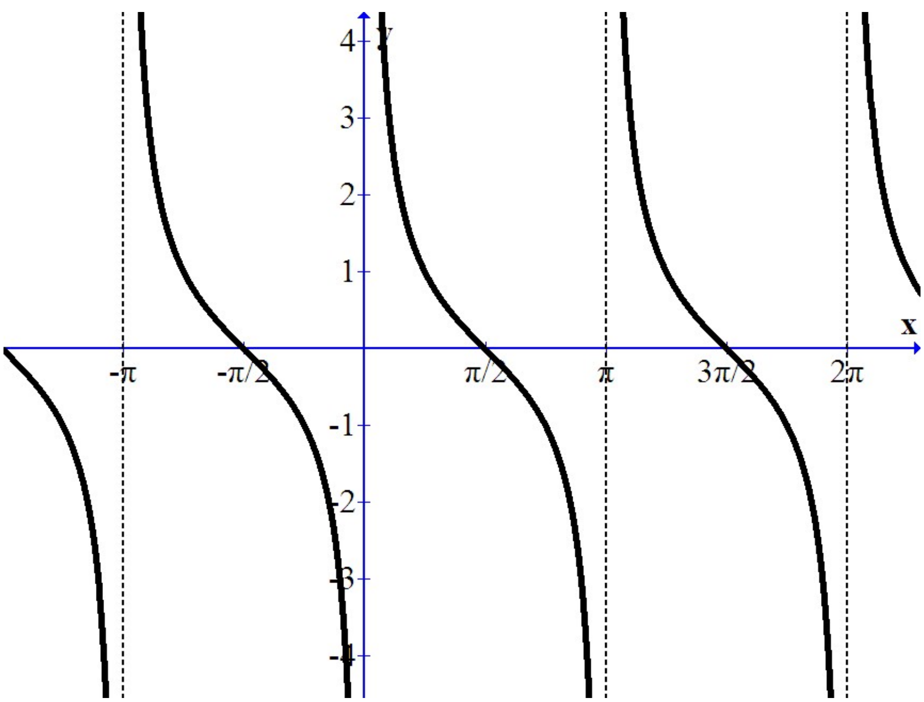
$y = \tan x$



- Domain: All Real Numbers except $x = (2k+1)\pi/2$ (Everything except odd multiples of $\pi/2$).
- Range: **R**, all real numbers.
- Period: $P = \pi$
- The Fundamental Cycle: $(-pi/2, \pi/2)$
- Symmetry: The graph of $y = \tan x$ is symmetric about the origin. The function $y = \tan x$ is an odd function. That is, $\tan(-x) = -\tan x$.

The Cotangent Graph

$y = \cot x$



- Domain: All Real Numbers except $x = k\pi$ (Everything except multiples of π).
- Range: **R**, all real numbers.

- Period: $P = \pi$
- The Fundamental Cycle: $(0, \pi)$
- Symmetry: The graph of $y = \cot x$ is symmetric about the origin. The function $y = \cot x$ is an odd function. That is, $\cot(-x) = -\cot x$.

Guidelines to Sketching the Tangent and Cotangent Graphs.

Step 1: Determine the period. Use $P = \pi/b$, where b is the coefficient of x.

Step2: Locate the vertical asymptotes.

- For $y = a \tan bx$, solve for $bx = -\pi/2$, $bx = \pi/2$, $bx = 3\pi/2$, etc.
- For $y = a \cot bx$, solve for $bx = 0$, $bx = \pi$, $bx = 2\pi$, etc.

Find the x-intercepts by finding the midpoint of the vertical asymptotes (add the two values and then divide by 2).

Step 3: Sketch three vertical asymptotes.

Step 4: Sketch the graph. The graph should look pretty much like the graph of the original function before any translations.

Step 5: Label the x-intercepts.

Example Sketch $y = \tan 2x$.

Example Sketch $y = -3\tan\frac{1}{2}x$.

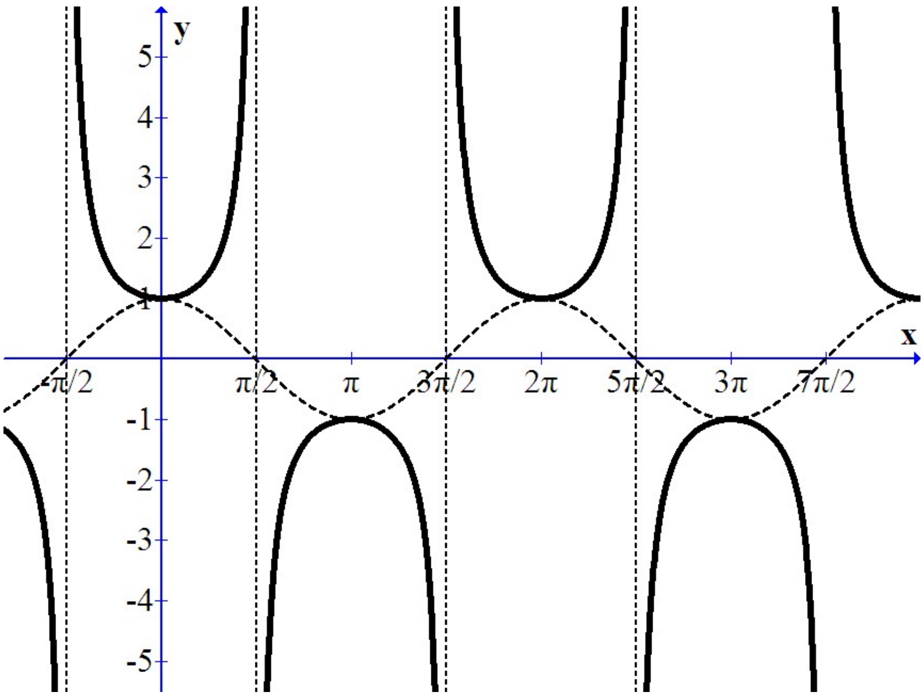
Example Sketch the graph of $y = \frac{1}{2}\cot 2x$.

Example Sketch the graph of $y = 2 + \tan x$.

Example Sketch the graph of $y = -2 - \cot(x - \pi/4)$.

The Secant Graph

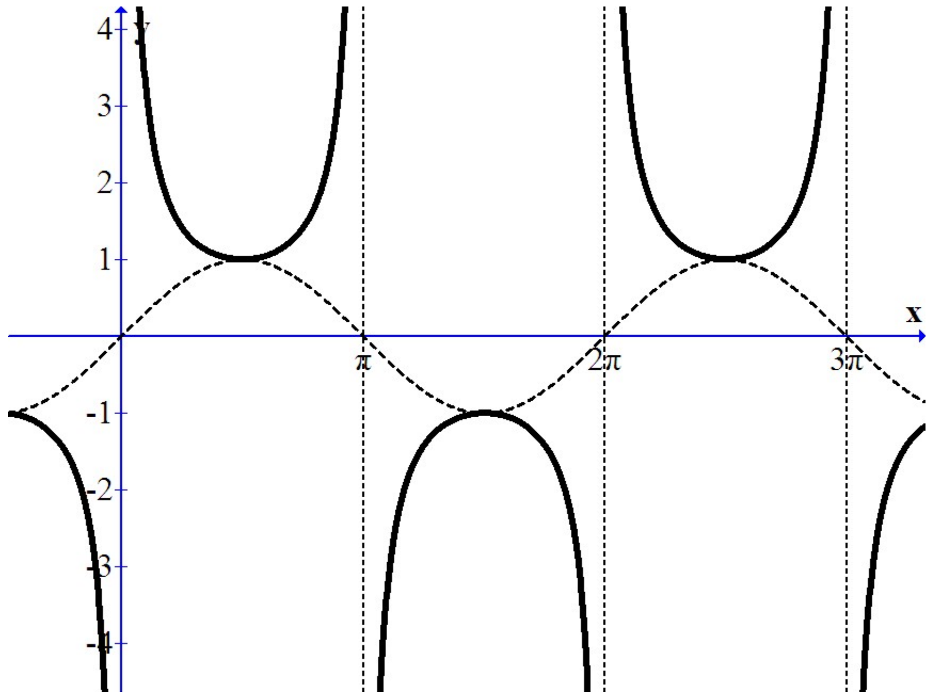
$$y = \sec x$$



- Domain: All Real Numbers except $x = (2k+1)\pi/2$ (Everything except odd multiples of $\pi/2$).
- Range: $y \leq -1$ or $1 \leq y$.
- Period: $P = 2\pi$
- Symmetry: The graph of $y = \sec x$ is symmetric about the y-axis. The function $y = \sec x$ is an even function. That is, $\sec(-x) = \sec x$.

The Cosecant Graph

$$y = \csc x$$



- Domain: All Real Numbers except $x = k\pi$ (Everything except multiples of π).
- Range: $y \leq -1$ or $1 \leq y$.
- Period: $P = 2\pi$
- Symmetry: The graph of $y = \csc x$ is symmetric about the y-axis. The function $y = \csc x$ is an even function. That is, $\csc(-x) = \csc x$.

Guidelines for Sketching the Graphs of Cosecant and Secant Functions

Step 1: Graph the corresponding reciprocal function as a guide. The co's don't go.

- To sketch $y = \csc x$, sketch $y = \sin x$.
- To sketch $y = \sec x$, sketch $y = \cos x$.

Step 2: Sketch the vertical asymptotes. The vertical asymptotes will be pass through the x-intercepts of the reciprocal functions.

Step 3: Sketch the graph of the function by making loops.

Example Sketch the graph of $y = 2\sec\frac{1}{2}x$.

Example Sketch the graph of $y = \frac{3}{2}\csc\left(x - \frac{\pi}{2}\right)$.