

7.3 Vectors

1. Write the vector in component form that has starting point at $P(-2, 7)$ and ending point at $(3, -1)$.

SOLUTION $\langle 3 - (-2), -1 - 7 \rangle = \langle 5, -8 \rangle$

2. Find the component form for each vector \mathbf{v} with the given magnitude and direction angle θ .

(a) $\|\mathbf{v}\| = 2, \theta = 30^\circ$

SOLUTION $\langle 2 \cos 30^\circ, 2 \sin 30^\circ \rangle = \langle 2(\sqrt{3}/2), 2(1/2) \rangle = \langle \sqrt{3}, 1 \rangle$

(b) $\|\mathbf{v}\| = 12, \theta = 120^\circ$

SOLUTION $\langle 12 \cos 120^\circ, 12 \sin 120^\circ \rangle = \langle 12(-1/2), 12(\sqrt{3}/2) \rangle = \langle -6, 6\sqrt{3} \rangle$

(c) $\|\mathbf{v}\| = 4, \theta = 210^\circ$

SOLUTION $\langle 4 \cos 210^\circ, 4 \sin 210^\circ \rangle = \langle 4(-\sqrt{3}/2), 4(-1/2) \rangle = \langle -2\sqrt{3}, -2 \rangle$

(d) $\|\mathbf{v}\| = 3, \theta = 315^\circ$

SOLUTION $\langle 3 \cos 315^\circ, 3 \sin 315^\circ \rangle = \langle 3(\sqrt{2}/2), 3(-\sqrt{2}/2) \rangle = \langle \frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2} \rangle$

3. Find the magnitude and direction angle of each vector. Find the angle in degrees. Round to the nearest tenth of a degree when necessary.

(a) $\mathbf{a} = \langle -1, \sqrt{3} \rangle$

SOLUTION $\|\mathbf{a}\| = \|\langle -1, \sqrt{3} \rangle\| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{4} = 2$.
 $\cos \theta = \frac{a_x}{\|\mathbf{a}\|} = \frac{-1}{2}, \theta$ is quadrant II. $\theta = 120^\circ$.

(b) $\mathbf{b} = \langle -1/2, -\sqrt{3}/2 \rangle$

SOLUTION $\|\mathbf{b}\| = \|\langle -1/2, -\sqrt{3}/2 \rangle\| = \sqrt{(-1/2)^2 + (-\sqrt{3}/2)^2} = 1$.
 $\cos \theta = \frac{b_x}{\|\mathbf{b}\|} = \frac{-1}{2}, \theta$ is quadrant III. $\theta = 240^\circ$.

(c) $\mathbf{c} = \langle -3, -3 \rangle$

SOLUTION $\|\mathbf{c}\| = \|\langle -3, -3 \rangle\| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$.
 $\cos \theta = \frac{c_x}{\|\mathbf{c}\|} = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}}$, θ is quadrant II. $\theta = 135^\circ$.

(d) $\mathbf{d} = \langle -3, 4 \rangle$

SOLUTION $\|\mathbf{d}\| = \|\langle -3, 4 \rangle\| = \sqrt{(-3)^2 + (4)^2} = 5$. $\cos \theta = \frac{d_x}{\|\mathbf{d}\|} = \frac{-3}{5}$, θ is quadrant II. $\theta = \cos^{-1}(-3/5) \approx 126.9^\circ$.

(e) $\mathbf{v} = \langle 12, -5 \rangle$

SOLUTION $\|\mathbf{v}\| = \|\langle 12, -5 \rangle\| = \sqrt{(12)^2 + (-5)^2} = 13$. $\cos \theta = \frac{v_x}{\|\mathbf{v}\|} = \frac{12}{13}$, θ is quadrant I. $\theta = \cos^{-1}(12/13) \approx 23.7^\circ$.

4. Let $\mathbf{r} = \langle 3, -2 \rangle$, $\mathbf{s} = \langle -1, 5 \rangle$, and $\mathbf{t} = \langle 4, -6 \rangle$. Perform the operations indicated.

(a) $-4\mathbf{s}$

SOLUTION $= -4\langle -1, 5 \rangle = \langle 4, -20 \rangle$

(b) $3\mathbf{r} - 2\mathbf{t}$

SOLUTION $= 3\langle 3, -2 \rangle - 2\langle 4, -6 \rangle = \langle 9, -6 \rangle + \langle -8, 12 \rangle = \langle 1, 6 \rangle$

(c) $-\mathbf{r} + 3\mathbf{s}$

SOLUTION $= -\langle 3, -2 \rangle + 3\langle -1, 5 \rangle = \langle -3, 2 \rangle + \langle -3, 15 \rangle = \langle -6, 17 \rangle$

(d) $2\mathbf{r} - \mathbf{s} - 4\mathbf{t}$

SOLUTION $= 2\langle 3, -2 \rangle - \langle -1, 5 \rangle - 4\langle 4, -6 \rangle = \langle -9, 15 \rangle$

(e) $\mathbf{s} \cdot \mathbf{r}$

SOLUTION $\langle -1, 5 \rangle \cdot \langle 3, -2 \rangle = (-1)(3) + (5)(-2) = -13$

(f) $\mathbf{s} \cdot \mathbf{t}$

SOLUTION $\langle -1, 5 \rangle \cdot \langle 4, -6 \rangle = (-1)(4) + (5)(-6) = -34$

(g) $\mathbf{r} \cdot \mathbf{t}$

SOLUTION $\langle 3, -2 \rangle \cdot \langle 4, -6 \rangle = (3)(4) + (-2)(-6) = 24$

5. Find the angle to the nearest tenth of a degree between each of given pair of vectors.

(a) $\langle 3, 4 \rangle, \langle -12, 5 \rangle$

SOLUTION

$$\begin{aligned}\cos \theta &= \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{\langle 3, 4 \rangle \cdot \langle -12, 5 \rangle}{\|\langle 3, 4 \rangle\| \|\langle -12, 5 \rangle\|} = \frac{-16}{(5)(13)} = -\frac{16}{65} \\ \theta &= \cos^{-1}(-16/65) \approx 104.25^\circ\end{aligned}$$

(b) $\langle -2, -5 \rangle, \langle 1, -3 \rangle$

SOLUTION

$$\begin{aligned}\cos \theta &= \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{\langle -2, -5 \rangle \cdot \langle 1, -3 \rangle}{\|\langle -2, -5 \rangle\| \|\langle 1, -3 \rangle\|} = \frac{13}{\sqrt{29}\sqrt{10}} \\ \theta &= \cos^{-1}\left(\frac{13}{\sqrt{29}\sqrt{10}}\right) \approx 40.2^\circ\end{aligned}$$

(c) $\langle 1, -5 \rangle, \langle 2, -2 \rangle$

SOLUTION

$$\begin{aligned}\cos \theta &= \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{\langle 1, -5 \rangle \cdot \langle 2, -2 \rangle}{\|\langle 1, -5 \rangle\| \|\langle 2, -2 \rangle\|} = \frac{12}{\sqrt{26}\sqrt{8}} \\ \theta &= \cos^{-1}\left(\frac{12}{\sqrt{26}\sqrt{8}}\right) \approx 33.7^\circ\end{aligned}$$

6. Determine whether each pair of vectors is parallel, perpendicular, or neither.

(a) $\langle 2, 3 \rangle, \langle 8, 12 \rangle$

SOLUTION $\langle 8, 12 \rangle = 4\langle 2, 3 \rangle$. They are parallel.

(b) $\langle 2, 3 \rangle, \langle -6, 4 \rangle$

SOLUTION $\langle 2, 3 \rangle \cdot \langle -6, 4 \rangle = 0$. They are perpendicular.

(c) $\langle 2, -4 \rangle, \langle 2, 1 \rangle$

SOLUTION $\langle 2, -4 \rangle \cdot \langle 2, 1 \rangle = 0$. They are perpendicular.

7. Write each vector as a linear combination of vectors \mathbf{i} and \mathbf{j} .

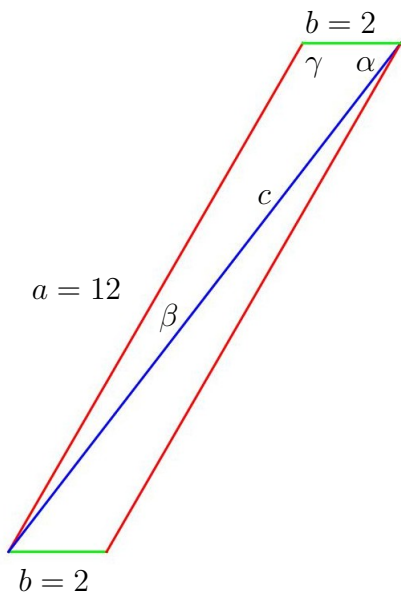
(a) $\langle 7, -3 \rangle$

SOLUTION $7\mathbf{i} - 3\mathbf{j}$

(b) $\langle 1, 5 \rangle$ SOLUTION $\mathbf{i} + 5\mathbf{j}$ (c) $\langle \sqrt{2}, -5 \rangle$ SOLUTION $\sqrt{2}\mathbf{i} - 5\mathbf{j}$

8. Find the magnitude of the resultant force and the angle between the resultant and each force, when forces of 2 lb and 12 lb act at an angle of 60° to each other.

SOLUTION



We can label the two vectors \mathbf{a} and \mathbf{b} . The length of the vector is $a = \|\mathbf{a}\| = 12$, and the length of the vector \mathbf{b} is $b = \|\mathbf{b}\| = 2$. The resultant is the vector $\mathbf{c} = \mathbf{a} + \mathbf{b}$. The length of the resultant is c . The angle opposite a is α , the angle opposite b is β , the angle opposite c is γ . We want to solve the triangle with sides a , b , and c .

First we need to find γ . Opposite angles of a parallelogram are congruent, and the sum of the angles of a quadrilateral is 360° . We have $2 \cdot 60^\circ + 2\gamma = 360$. Therefore, $\gamma = 120^\circ$.

We find the length of the resultant, c , using the Law of Cosines.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos \gamma \\ c &= \sqrt{a^2 + b^2 - 2ab \cos \gamma} = \sqrt{12^2 + 2^2 - 2 \cdot 12 \cdot 2 \cos 120^\circ} \\ c &= \sqrt{144 + 4 - 48(-1/2)} = \sqrt{172} = \sqrt{4 \cdot 43} = 2\sqrt{43} \approx 13.115 \end{aligned}$$

We can find α by using the Law of Sines.

$$\frac{\sin \alpha}{a} = \frac{\sin \gamma}{c}$$

$$\begin{aligned}\sin \alpha &= \frac{a \sin \gamma}{c} = \frac{12 \sin 120^\circ}{2\sqrt{43}} = \frac{12 \cdot (\sqrt{3}/2)}{2\sqrt{43}} = \frac{3\sqrt{3}}{\sqrt{43}} \\ \alpha &= \sin^{-1} \left(\frac{3\sqrt{3}}{\sqrt{43}} \right) \approx 52.4^\circ\end{aligned}$$

We can now find β . We have

$$\beta = 180 - 120^\circ - 52.4^\circ = 7.6^\circ$$

In summary, the magnitude of the resultant is approximately 13.1 lb, and the angle that the resultant makes with each force is 52° and 7.6° .