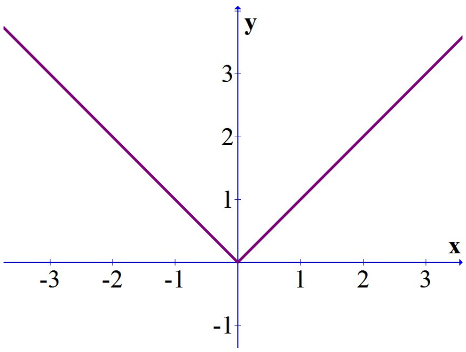
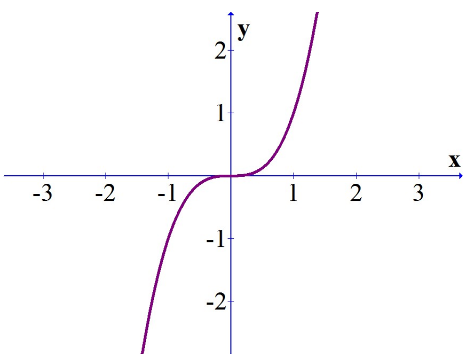
Use the horizontal line test to determine whether the function whose graph is shown is one-to-one.

1.




2.




Determine whether the function is one-to-one using the definition of one-to-one.

**Example** $f(x) = 2x + 3$

SOLUTION Let $f(a) = f(b)$. Show that $a = b$.

$$\begin{aligned} f(a) &= f(b) \\ 2a + 3 &= 2b + 3 \\ 2a &= 2b \\ a &= b \end{aligned}$$

Therefore, $f$ is one-to-one.

3. $h(x) = 4x - 9$

4. $f(x) = \frac{1}{2}x + 1$

5. $\dfrac{x+2}{x+3}$

6. $\dfrac{x-1}{x+2}$

Find the inverse function using the switch and solve method.

7. $f(x) = -2x + 5$

8. $f(x) = 5x + 2$

9. $f(x) = \sqrt{3x-1}$

10. $f(x) = \dfrac{2x-1}{x-6}$

11. $f(x) = \dfrac{x+2}{x-3}$

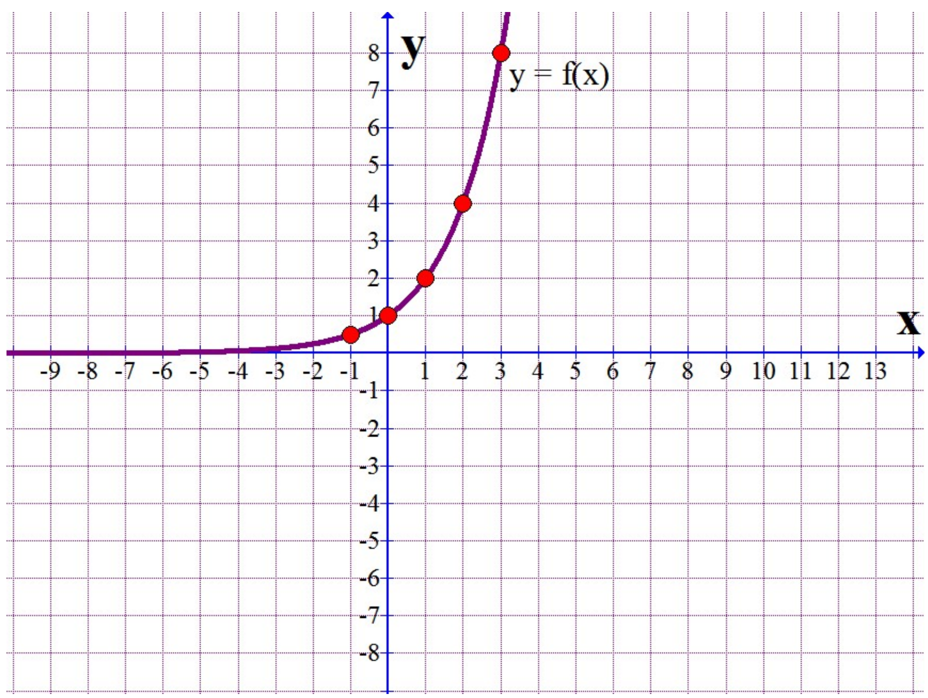
Find $f \circ g(x)$ and $g \circ f(x)$. Then determine whether $f$ and $g$ are inverse functions of each other.

12. $f(x) = \dfrac{1}{x} + 3, \quad g(x) = \dfrac{1}{x-3}$

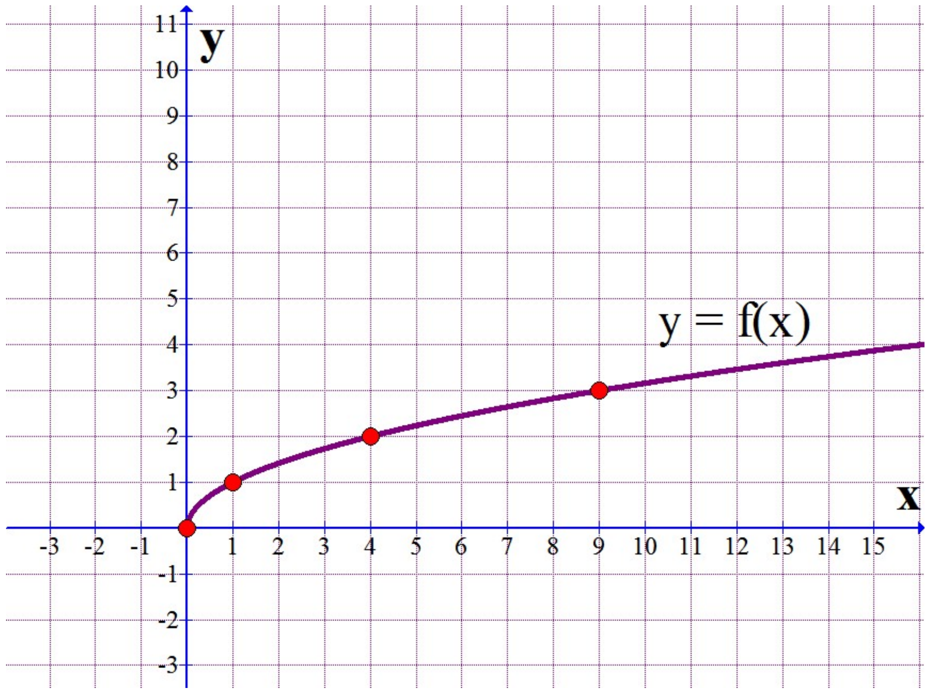
13. $f(x) = \sqrt[3]{\dfrac{x-2}{5}}, \quad g(x) = 5x^3 + 2$

The graph of $f$ is given. Sketch the graph of $f^{-1}$.

14.

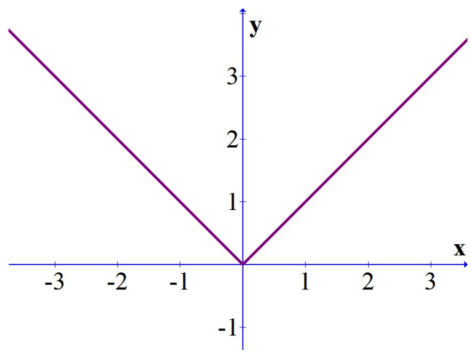



15.


y
x
y = f(x)
11
10
9
8
7
6
5
4
3
2
1
-1
-2
-3
-3 -2 -1 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

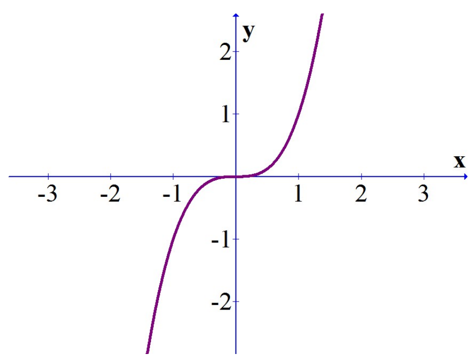
Use the horizontal line test to determine whether the function whose graph is shown is one-to-one.

1.




The function is not one-to-one because it fails the horizontal line test.

2.




The function is one-to-one because it passes the horizontal line test.

Determine whether the function is one-to-one using the definition of one-to-one.

**Example** $f(x) = 2x + 3$

Solution Let $f(a) = f(b)$. Show that $a = b$.

$$\begin{aligned} f(a) &= f(b) \\ 2a + 3 &= 2b + 3 \\ 2a &= 2b \\ a &= b \end{aligned}$$

Therefore, $f$ is one-to-one.

3. $h(x) = 4x - 9$

SOLUTION Let $f(a) = f(b)$. Show that $a = b$.

$$\begin{aligned} f(a) &= f(b) \\ 4a - 9 &= 4b - 9 \\ 4a &= 4b \\ a &= b \end{aligned}$$

Therefore, $f$ is one-to-one.

4. $f(x) = \frac{1}{2}x + 1$

SOLUTION Let $f(a) = f(b)$. Show that $a = b$.

$$\begin{aligned} f(a) &= f(b) \\ \frac{1}{2}a + 1 &= \frac{1}{2}b + 1 \\ \frac{1}{2}a &= \frac{1}{2}b \\ a &= b \end{aligned}$$

Therefore, $f$ is one-to-one.

5. $\dfrac{x+2}{x+3}$

SOLUTION Let $f(a) = f(b)$. Show that $a = b$.

$$\begin{aligned} f(a) &= f(b) \\ \frac{a+2}{a+3} &= \frac{b+2}{b+3} \\ (a+2)(b+3) &= (b+2)(a+3) \\ ab + 3a + 2b + 6 &= ab + 3b + 2a + 6 \\ 3a + 2b &= 3b + 2a \\ 3a - 2a &= 3b - 2b \\ a &= b \end{aligned}$$

Therefore, $f$ is one-to-one.

6. $\dfrac{x-1}{x+2}$

   Solution Let $f(a) = f(b)$. Show that $a = b$.

$$\begin{aligned}
f(a) &= f(b) \\
\frac{a-1}{a+2} &= \frac{b-1}{b+2} \\
(a-1)(b+2) &= (b-1)(a+2) \\
ab + 2a - b - 2 &= ab + 2b - a - 2 \\
2a - b &= 2b - a \\
2a + a &= 2b + b \\
3a &= 3b \\
a &= b
\end{aligned}$$

Therefore, $f$ is one-to-one.

Find the inverse function using the switch and solve method.

7. $f(x) = -2x + 5$

   Solution

$$\begin{aligned}
y &= -2x + 5 \\
x &= -2y + 5 \;\; <\text{—Switch} \\
x - 5 &= -2y \;\; <\text{—Solve} \\
y &= -\frac{1}{2}x + \frac{5}{2} \\
f^{-1}(x) &= -\frac{1}{2}x + \frac{5}{2}
\end{aligned}$$

8. $f(x) = 5x + 2$

   Solution

$$\begin{aligned}
y &= 5x+2\\
x &= 5y+2\\
x-2 &= 5y\\
y &= \frac{1}{5}x-\frac{2}{5}\\
f^{-1}(x) &= \frac{1}{5}x-\frac{2}{5}
\end{aligned}$$

9. $f(x)=\sqrt{3x-1}$

   Solution

$$\begin{aligned}
y &= \sqrt{3x-1}, \quad y \geq 0\\
x &= \sqrt{3y-1}, \quad x \geq 0\\
x^2 &= 3y-1, \quad x \geq 0\\
3y &= x^2+1, \quad x \geq 0\\
y &= \frac{x^2+1}{3}, \quad x \geq 0\\
f^{-1}(x) &= \frac{x^2+1}{3}, \quad x \geq 0
\end{aligned}$$

10. $f(x)=\dfrac{2x-1}{x-6}$

    Solution

$$\begin{aligned}
y &= \frac{2x-1}{x-6}\\
x &= \frac{2y-1}{y-6}\\
x(y-6) &= 2y-1\\
xy-6x &= 2y-1
\end{aligned}$$

$$\begin{aligned} xy - 2y &= 6x - 1 \\ y(x-2) &= 6x - 1 \\ y &= \frac{6x-1}{x-2} \\ f^{-1}(x) &= \frac{6x-1}{x-2} \end{aligned}$$

11. $f(x) = \dfrac{x+2}{x-3}$

SOLUTION

$$\begin{aligned} y &= \frac{x+2}{x-3} \\ x &= \frac{y+2}{y-3} \\ x(y-3) &= y+2 \\ xy - 3x &= y+2 \\ xy - y &= 3x+2 \\ y(x-1) &= 3x+2 \\ y &= \frac{3x+2}{x-1} \\ f^{-1}(x) &= \frac{3x+2}{x-1} \end{aligned}$$

Find $f \circ g(x)$ and $g \circ f(x)$. Then determine whether $f$ and $g$ are inverse functions of each other.

12. $f(x) = \dfrac{1}{x} + 3, \quad g(x) = \dfrac{1}{x-3}$

SOLUTION

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{1}{x-3}\right) \\
&= \frac{1}{\frac{1}{x-3}} + 3 \\
&= (x-3) + 3 = x
\end{aligned}
\qquad
\begin{aligned}
g(f(x)) &= g\left(1/x + 3\right) \\
&= \frac{1}{(1/x+3) - 3} \\
&= \frac{1}{1/x} = x
\end{aligned}
$$

Yes, the functions are inverses.

13. $f(x) = \sqrt[3]{\dfrac{x-2}{5}}, \quad g(x) = 5x^3 + 2$

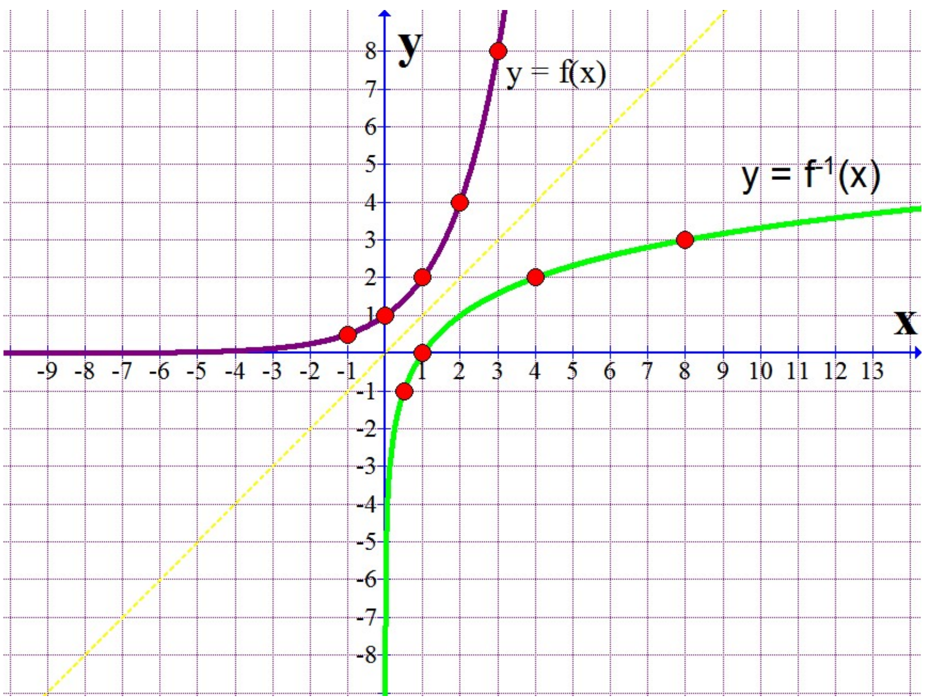
SOLUTION

$$
\begin{aligned}
g(f(x)) &= g\left(\sqrt[3]{\frac{x-2}{5}}\right) \\
&= 5\left(\sqrt[3]{\frac{x-2}{5}}\right)^3 + 2 \\
&= 5\frac{(x-2)}{5} + 2 = (x-2) + 2 = x
\end{aligned}
$$

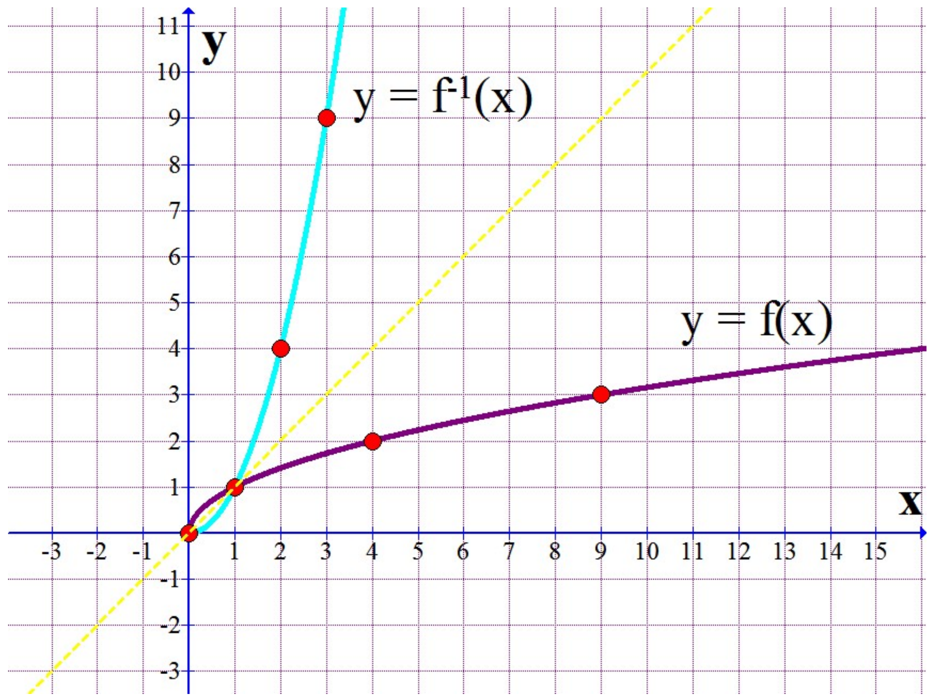
Yes, the functions are inverses.

The graph of $f$ is given. Sketch the graph of $f^{-1}$.

14. SOLUTION




15. SOLUTION


y
x
y = f-1(x)
y = f(x)