Math 180 Monday 2/08

9:35-11:50am section.

Quiz Thurs. 2/11
Sections 2.2, 2.3, 2.4

Practice Problems

§2.4

(1) Use the precise definition of limit to prove

$$\lim_{x \to 3} (-2x+7) = 1$$

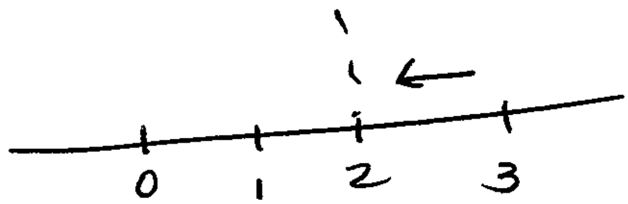
<u>Proof</u> Let $\varepsilon > 0$, choose $\delta = \frac{\varepsilon}{2}$

if $0 < |x-3| < \delta$,

then
$$\begin{aligned} |f(x) - L| &= |(-2x+7) - 1| \\ &= |-2x+6| \\ &= |-2(x-3)| \\ &= 2|x-3| \\ &< 2\delta = 2\left(\frac{\varepsilon}{2}\right) = \varepsilon \quad \square \end{aligned}$$

§2.2 Evaluate the infinite limit.

(2) $\lim_{x \to 2^+} \frac{3x}{x-2} = \boxed{\infty}$ $\frac{6}{0}$ form



Test number $x = 2.01$

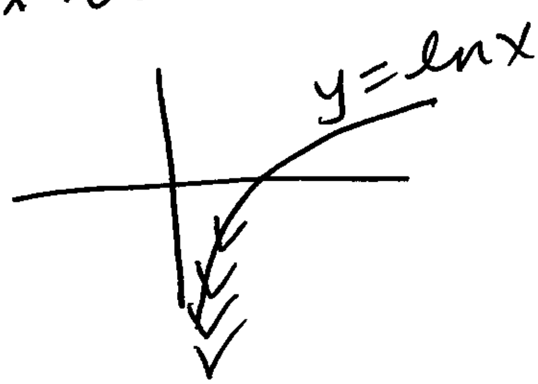
$y = \frac{3(2.01)}{(2.01-2)} = \frac{6.03}{.01}$

$= 603$

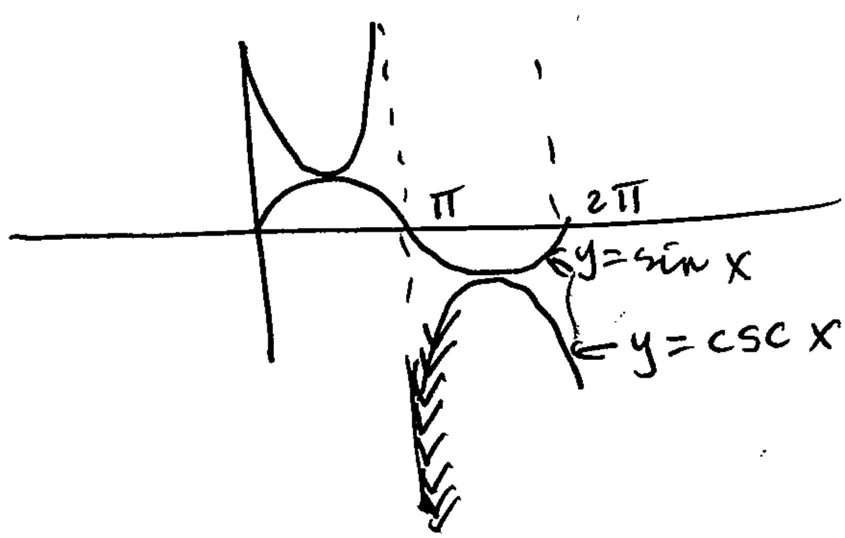
big pos

(2) $\lim_{x \to 0^+} \ln x = \boxed{-\infty}$

These problems can also be done by plotting points.



(3) $\lim_{x \to \pi^+} \csc x = \boxed{-\infty}$ $y = \csc x = \frac{1}{\sin x}$



§2.3 Evaluate the limit.

(5) $\lim\limits_{x \to -2} \frac{x^2+6x+8}{x^3+8}$ $\frac{0}{0}$ form

Use:
$a^3+b^3=(a+b)(a^2-ab+b^2)$

$= \lim\limits_{x \to -2} \frac{\cancel{(x+2)}(x+4)}{\cancel{(x+2)}(x^2-2x+4)}$

$= \lim\limits_{x \to -2} \frac{(x+4)}{(x^2-2x+4)} = \frac{(-2+4)}{((-2)^2-2(-2)+4)}$

$= \frac{2}{4+4+4} = \frac{2}{12} = \frac{\cancel{2}}{\cancel{12}} = \frac{1}{6}$

(6) $\lim\limits_{x \to 0} \frac{\sqrt{x^2+16}-4}{x^2}$ $\frac{0}{0}$ form

$= \lim\limits_{x \to 0} \left(\frac{\sqrt{x^2+16}-4}{x^2}\right)\left(\frac{\sqrt{x^2+16}+4}{\sqrt{x^2+16}+4}\right)$

$= \lim\limits_{x \to 0} \frac{(x^2+\cancel{16})-\cancel{16}}{x^2(\sqrt{x^2+16}+4)} = \lim\limits_{x \to 0} \frac{\cancel{x^2}}{\cancel{x^2}(\sqrt{x^2+16}+4)}$

$= \frac{1}{\sqrt{0^2+16}+4} = \frac{1}{4+4} = \frac{1}{8}$

7 $\lim_{x \to 6} \dfrac{\frac{1}{x} - \frac{1}{6}}{x-6}$ $\quad \frac{0}{0}$ form

$= \lim_{x \to 6} \dfrac{\left(\frac{1}{x} - \frac{1}{6}\right) 6x}{(x-6)\, 6x}$

$= \lim_{x \to 6} \dfrac{(6-x)}{(x-6)(6x)}$ $\quad \leftarrow$ use $a-b = -(b-a)$

$= \lim_{x \to 6} \dfrac{-\cancel{(x-6)}}{\cancel{(x-6)}(6x)}$

$= \lim_{x \to 6} \dfrac{-1}{6x} = \dfrac{-1}{6(6)} = -\dfrac{1}{36}$

Homework Review

§2.3 #43

$$\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$

if $x<0$, then $|x| = -x$

$x<0$ $\quad 0$

x is neg

$$= \lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{(-x)} \right) = \lim_{x \to 0^-} \left(\frac{1}{x} + \frac{1}{x} \right)$$

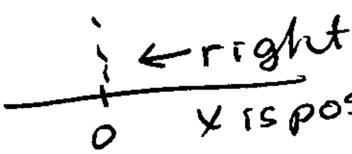
$$= \lim_{x \to 0^-} \frac{2}{x} \qquad \frac{2}{0} \text{ form}$$

$\boxed{\text{DNE}}$

$$= -\infty$$

#44

$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$



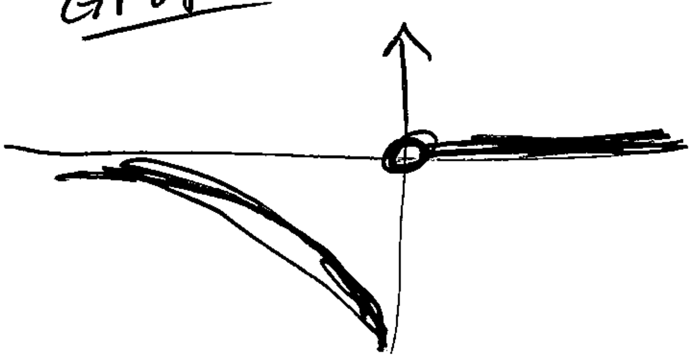
x is pos

if $x>0$, then $|x| = x$

$$= \lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x} \right) = \lim_{x \to 0} (0) = \boxed{0}$$

Extra:

Graph: $y = \frac{1}{x} - \frac{1}{|x|} = \begin{cases} 0 & \text{if } x>0 \\ 2/x & \text{if } x<0 \\ \text{undef} & \text{if } x=0 \end{cases}$



§2.3 #37 Prove using the Squeeze Theorem that (Sandwich Theorem)

$$\lim_{x\to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$$

<u>Proof:</u> ① Inequality (Make the sandwich)

$$-1 \le \cos\left(\frac{2}{x}\right) \le 1$$

Multiply through by x^4

$$-x^4 \le x^4 \cos\left(\frac{2}{x}\right) \le x^4$$

② Limits of Outside Functions should be equal. (Find limits of the bread)

- $\lim_{x\to 0} -x^4 = -(0)^4 = 0$
- $\lim_{x\to 0} x^4 = 0^4 = 0.$

Therefore by the Squeeze Theorem

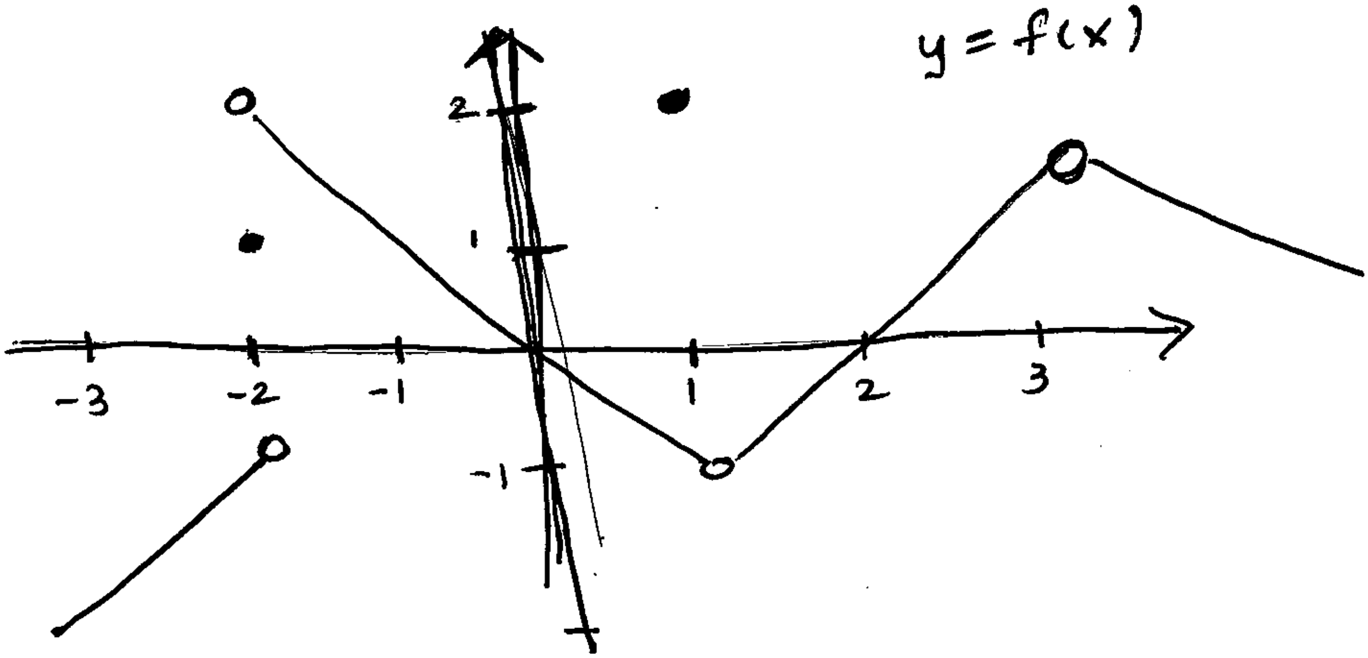
$$\lim_{x\to 0} x^4 \cos\left(\frac{2}{x}\right) = 0 \qquad \square$$

Notes

§2.5 Continuity

HW §2.5 #3-6, 15-20, 31-34, 37-39, 47-50.

Example



Informal Definition: A function f is continuous at a number a, if we do not have to pick up the pen when we sketch the graph. If we have to pick up the pencil at a, then it is called discontinuous at a.

- List the values of a where f is discontinuous.

Solution: $a = -2$, $a = 1$, $a = 3$

Three Part Definition of Continuity (Memorize for the Test)

A function f is continuous at a number a if

1. $f(a)$ is defined. (exists)
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Refering to the graph of f on the ~~last~~ previous page, state where f is discontinuous and which requirement fails.

Solution:

- $a = -2$ $\qquad \lim_{x \to -2} f(x)$ DNE
- $a = 1$ $\qquad \lim_{x \to 1} f(x) \neq f(1)$
- $a = 3$ $\qquad f(3)$ DNE

§2.3 #38 11/10 - 12:25 am

Prove using the Squeeze Theorem (Sandwich Theorem) that

$$\lim_{x\to 0^+} \sqrt{x}\, e^{\sin(\pi/x)} = 0$$

Proof: 1. Inequality (Make the sandwich)

$$-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$$

$$e^{-1} \le e^{\sin(\pi/x)} \le e^{1}$$

Multiply through by $\sqrt{x}$

$$e^{-1}\sqrt{x} \le \sqrt{x}\, e^{\sin(\pi/x)} \le e\sqrt{x}$$

2. Limit of outside Functions (bread).

- $\lim_{x\to 0^+} e^{-1}\sqrt{x} = e^{-1}\sqrt{0} = 0$
- $\lim_{x\to 0^+} e\sqrt{x} = e\sqrt{0} = 0$

Conclusion: Therefore by Sq. Thm

$$\lim_{x\to 0} \sqrt{x}\, e^{\sin(\pi/x)} = 0.$$ $\square$

12:45-2pm Section

§2.4 #19 Prove using the definition of limit

$$\lim_{x \to 3} \frac{x}{5} = \frac{3}{5}$$

Proof: Let $\varepsilon > 0$ choose $\delta = 5\varepsilon$

if $0 < |x-3| < \delta$

then

$$|f(x) - L| = \left| \frac{x}{5} - \frac{3}{5} \right|$$

$$= \left| \frac{1}{5}(x-3) \right|$$

$$= \frac{1}{5}|x-3|$$

$$< \frac{1}{5}\delta = \frac{1}{5}(5\varepsilon) = \varepsilon \quad \square$$