Math 180 Tues. Feb. 9, 2010

Quiz Thurs. 2/11
§ 2.2, 2.3, 2.4

Practice Problems.

1. § 2.4 Use the precise definition of limit to prove that

$$\lim_{x \to 3} \left(\frac{x}{2}+1\right) = \frac{5}{2}$$

<u>Proof</u>: Let $\varepsilon > 0$, choose $\delta = 2\varepsilon$

if $0 < |x-3| < \delta$

then $|f(x) - L|$

$$= \left|\left(\frac{x}{2}+1\right) - \frac{5}{2}\right|$$

$$= \left|\frac{x}{2} + \frac{2}{2} - \frac{5}{2}\right|$$

$$= \left|\frac{x}{2} - \frac{3}{2}\right|$$

$$= \left|\frac{1}{2}(x-3)\right|$$

$$= \frac{1}{2}|x-3|$$

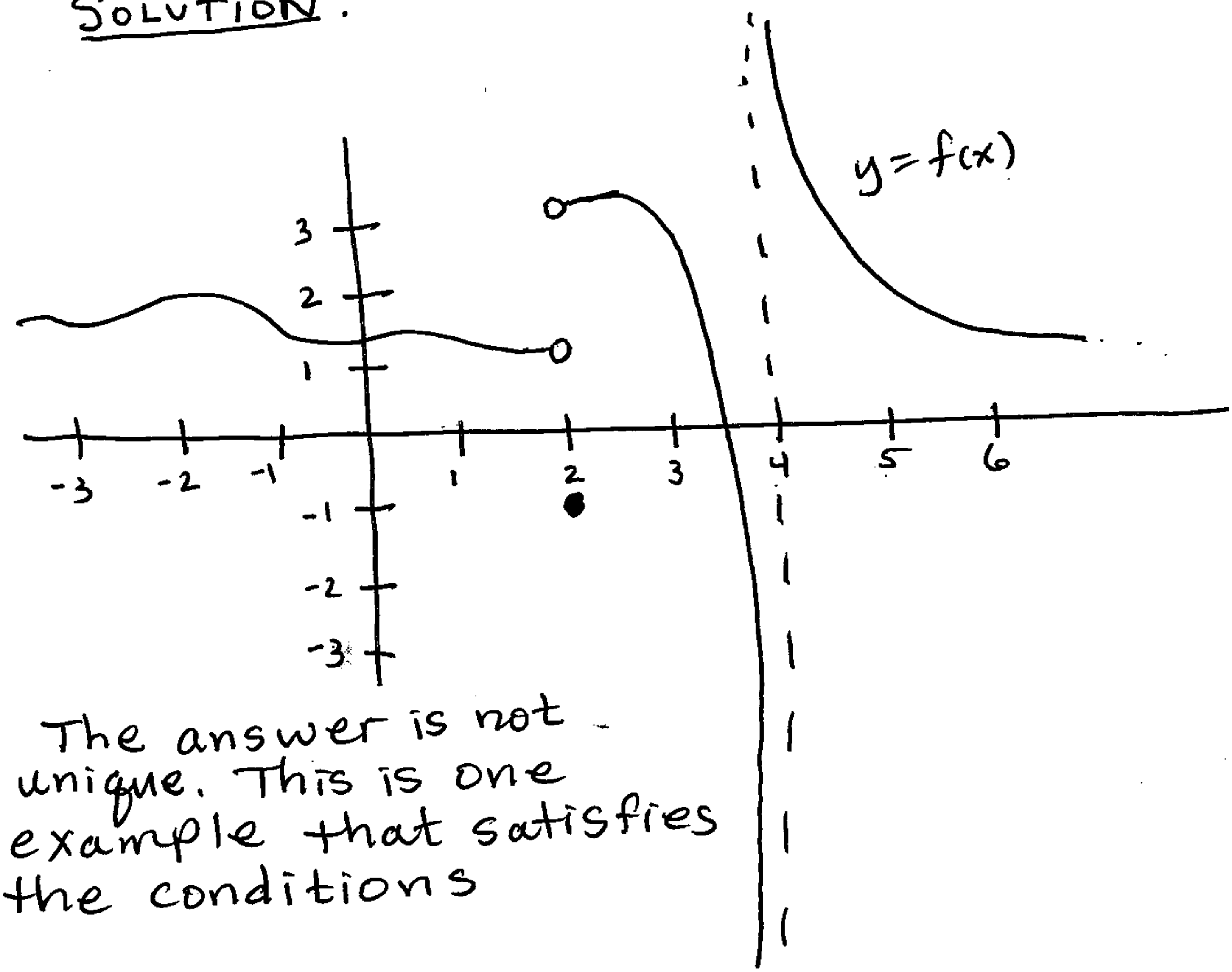
$$< \frac{1}{2}\delta = \frac{1}{2}(2\varepsilon) = \varepsilon \qquad \square$$

Practice Problems.

2. §2.2 Sketch the graph of an example of a function f that satisfies all of the given conditions.

- $\lim_{x \to 2^-} f(x) = 1$
- $\lim_{x \to 2^+} f(x) = 3$
- $f(2) = -1$
- $\lim_{x \to 4^-} f(x) = -\infty$
- $\lim_{x \to 4^+} f(x) = \infty$
- $f(4)$ is undefined
- f is continuous for all real numbers except $x=2$ and $x=4$

SOLUTION:



The answer is not unique. This is one example that satisfies the conditions

§2.5 Continuity (Continued)

Theorem

A function f is said to be continuous on an interval if it is continuous for all values on the interval.

<u>Theorem</u>: If f and g are continuous at a, and c is a constant, the the following functions are also continuous at a.

1. $f+g$
2. $f-g$
3. cf
4. fg
5. $\frac{f}{g}$ provided $g(a) \neq 0$.

<u>Theorem</u>: If f and g are continuous functions, then $y=f(g(x))$ is also continuous.

<u>Theorem</u> The following types of functions are continuous on their domain.

- polynomial
- rational
- root
- trigonometric
- inverse trig functions
- exponential
- log
- absolute value.

<u>Example</u>: Find the limits.

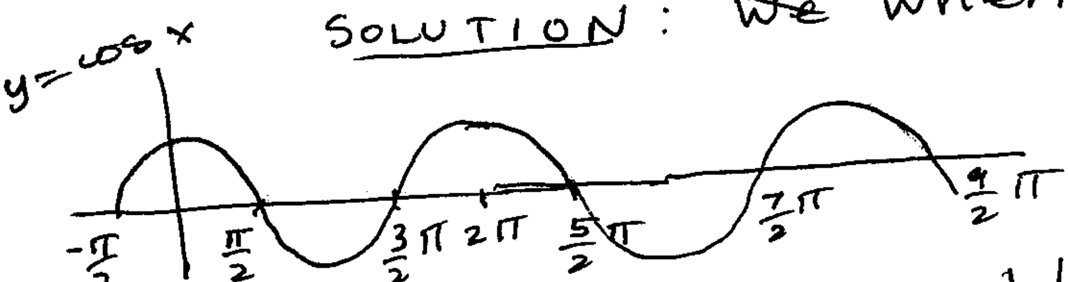
① $\lim_{x \to 0} \ln(\cos x)$

<u>SOLUTION</u> The function $y = \ln(\cos x)$ is continuous, so we put in directly $x = 0$.

$$\ln(\cos(0)) = \ln(1) = 0$$

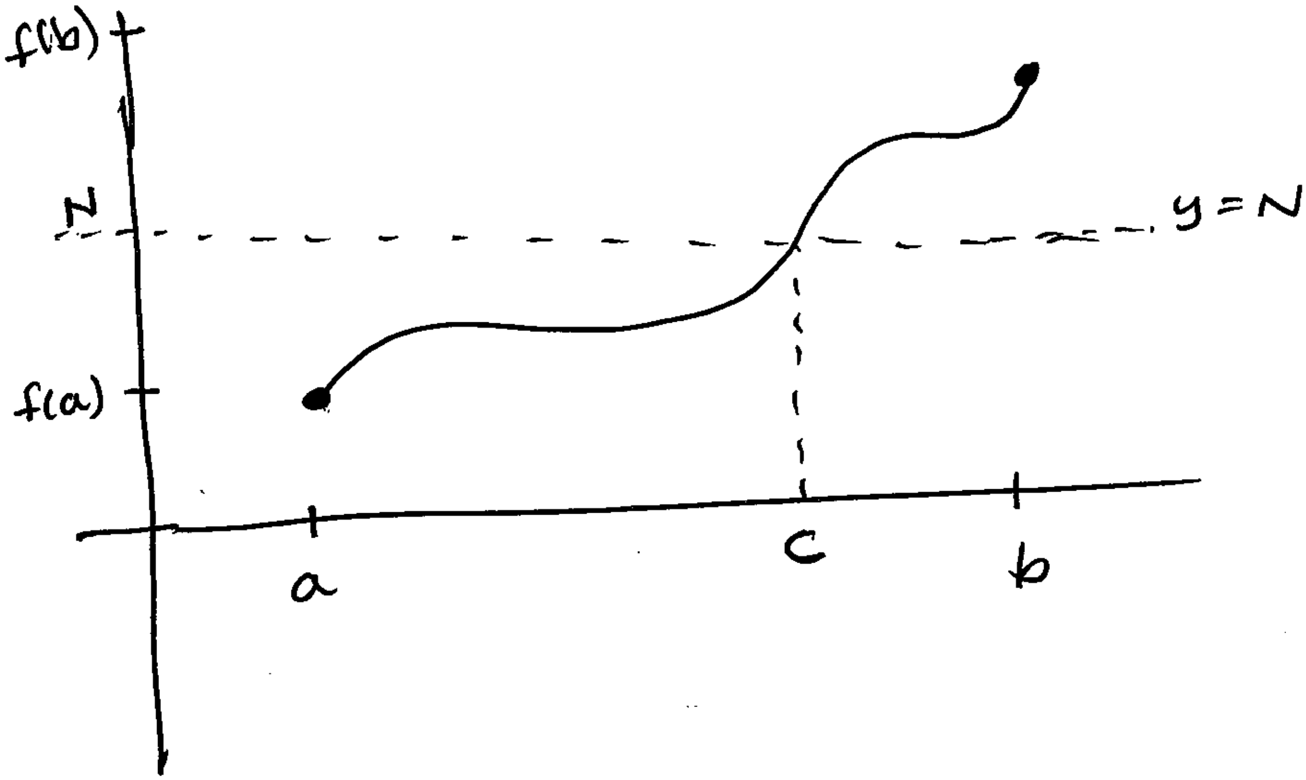
<u>Follow up</u>: On which intervals is $y = \ln(\cos x)$ continuous?

<u>SOLUTION</u>: ~~We~~ When $\cos x > 0$

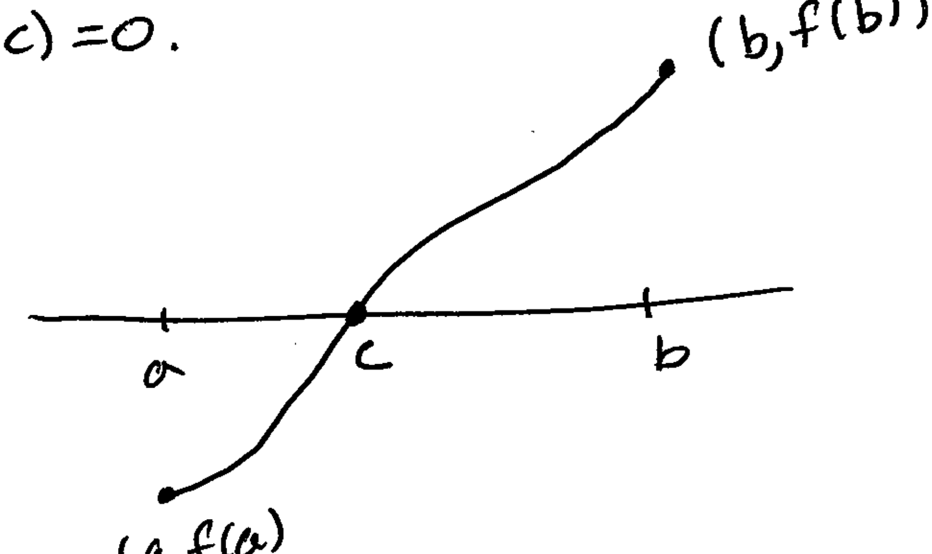


Answer $\displaystyle\bigcup_{\substack{n \text{ is} \\ \text{an} \\ \text{integer}}} \left(\left(\frac{4n-1}{2}\right)\pi, \left(\frac{4n+1}{2}\right)\pi \right)$

The Intermediat Value Theorem: Suppose that f is continuous on a closed interval $[a,b]$ and let N be any real number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in the open interval (a,b) such that $f(c)=N$.



<u>Corollary</u> If f is continuous on the interval $[a, b]$, and $f(a)$ and $f(b)$ are both nonzero and have opposite signs (one is positive, the other negative), then there is a number c in (a, b) such that $f(c) = 0$.



<u>Example</u>: show that the equation has at least one real root in the given interval.

$$4x^3 - 6x^2 + 3x - 2 = 0 \quad \text{on } (0, 2).$$

<u>SOLUTION</u> $f(x) = 4x^3 - 6x^2 + 3x - 2$ is continuous on $[0, 2]$ because it is a polynomial.

Endpoints $f(0) = -2$

$$f(2) = 4(2)^3 - 6(2)^2 + 3(2) - 2$$
$$= 12$$

So $f(0)$ is negative and $f(2)$ is positil

So there exists c in $(0,2)$ $0<c<2$

such that $f(c) = 0$.

by the Intermediate Value Theorem

Example: Let $f(x) = x^5 + 7x^4 + 5x^3 + 2x^2 + 9x - 10$.

A table of plotted points is shown below.

x	$f(x)$
-8	-6610
-7	-1690
-6	224
-5	620
-4	434
-3	170
-2	20
-1	-16
0	-10
1	14
2	200
3	980

Which intervals must contain a zero of f?

$[-8,-7]$, $[-7,-6]$, $[-6,-5]$, $[~~-4~~-5,-4]$, $[-4,-3]$, $[-3,-2]$, $[-2,-1]$, $[-1,0]$, $[0,1]$, $[1,2]$, $[2,3]$