Math 180 Weds 2/17/2010.

Practice Problems. (Chapter 2 Test Review)

① Use the precise definition of limit to prove that

$$\lim_{x \to -5} (3x+2) = -13.$$

SOLUTION Let $\varepsilon > 0$, choose $\delta = \frac{\varepsilon}{3}$

if $0 < |x - (-5)| < \delta$

then $|f(x) - L|$

$= |(3x+2) - (-13)|$

$= |3x+15|$

$= |3(x+5)|$

$= 3|x+5| < 3\delta = 3\left(\frac{\varepsilon}{3}\right) = \varepsilon$ Π

② Let $f(x) = x^2+5x-2$.

a) Find $f'(2)$ using the definition of derivative

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$f'(2) = \lim_{x \to 2} \frac{(x^2+5x-2) - (2^2+5(2)-2)}{x-2}$$

$$= \lim_{x \to 2} \frac{x^2+5x-14}{x-2} = \lim_{x \to 2} \frac{(x+7)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 2+7 = 9$$

1

b) Find the tangent line to f at $(2, 12)$.

$m = 9$

$x_1 = 2, \; y_1 = 12$

$y - y_1 = m(x - x_1)$

$y - 12 = 9(x - 2)$

$y - 12 = 9x - 18$

$\boxed{y = 9x - 6}$

(3) Sketch the graph of a function that satisfies

- $\lim_{x \to (-2)^-} f(x) = \infty$

  $\lim_{x \to (-2)^+} f(x) = -\infty$

  $f(-2)$ DNE

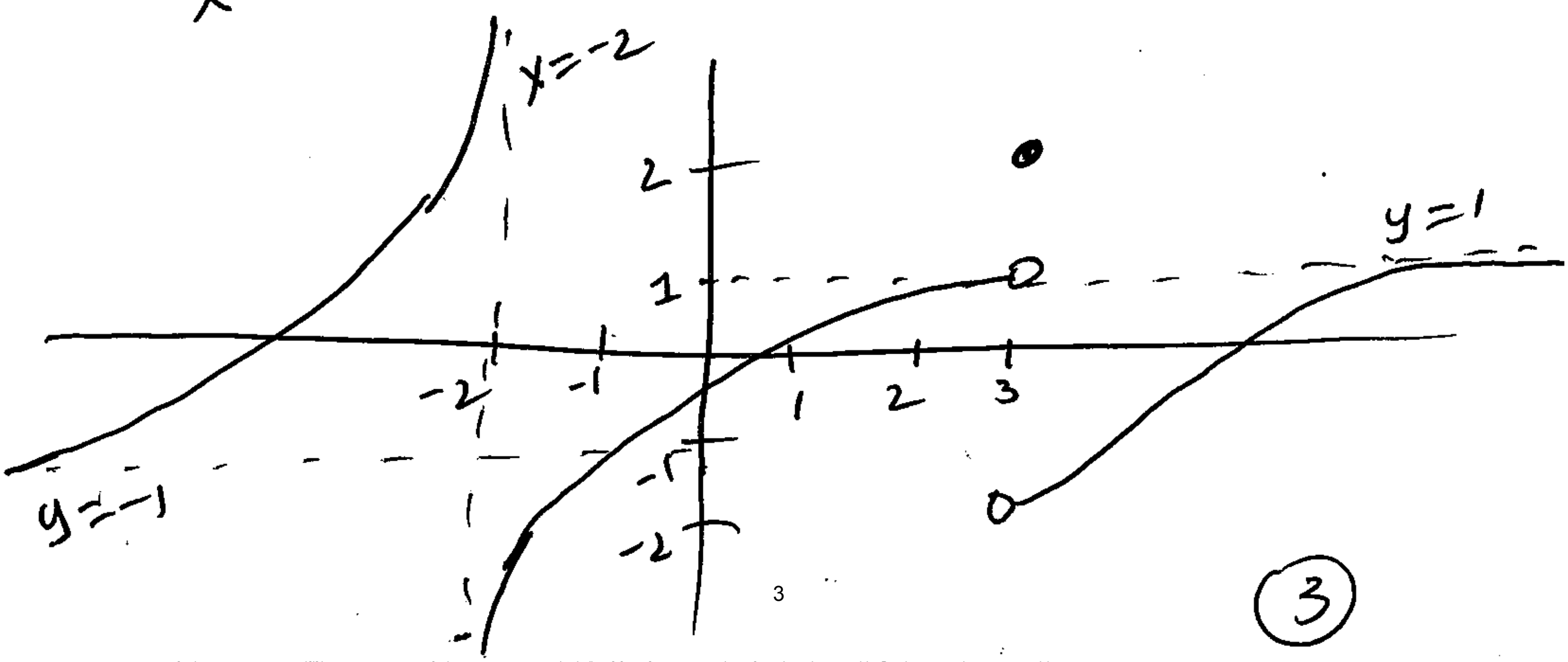
- $\lim_{x \to 3^-} f(x) = 1$

  $\lim_{x \to 3^+} f(x) = -2$

  $f(3) = 2$

- $\lim_{x \to \infty} f(x) = 1$

  $\lim_{x \to -\infty} f(x) = -1$



(3)

(4) a) State the three part definition of continuity of a function f at a number a.

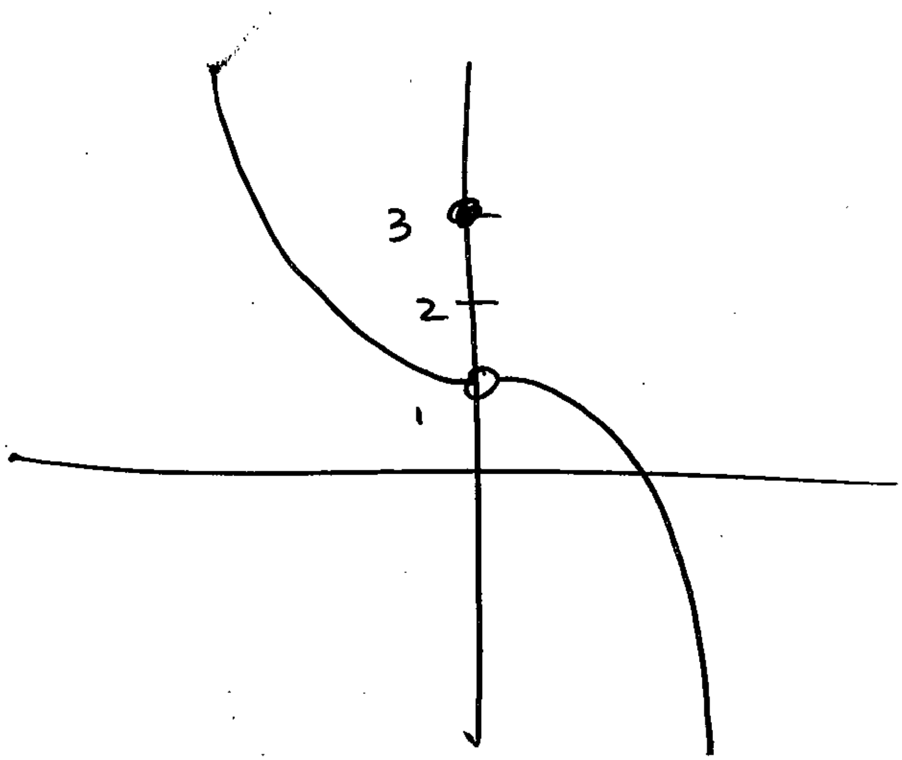
(i) $f(a)$ is defined

(ii) $\lim_{x \to a} f(x)$ exists

(iii) $\lim_{x \to a} f(x) = f(a)$

b) Let $f(x) = \begin{cases} e^{-x} & \text{if } x < 0 \\ 3 & \text{if } x = 0 \\ -x^2+1 & \text{if } x > 0 \end{cases}$

• sketch f



4C State where f is discontinuous. Which requirement(s) from the definition fail.

SOLUTION

$x = 0$

$$\lim_{x \to 0} f(x) \neq f(0)$$

(5) Squeeze Theorem. Suppose that

$$9x - 6 \leq f(x) \leq x^2 + 5x - 2.$$

Find $\lim_{x \to 2} f(x)$.

SOLUTION

$$\lim_{x \to 2} (9x - 6) = 9(2) - 6 = 12$$

$$\lim_{x \to 2} (x^2 + 5x - 2) = (2)^2 + 5(2) - 2 = 12$$

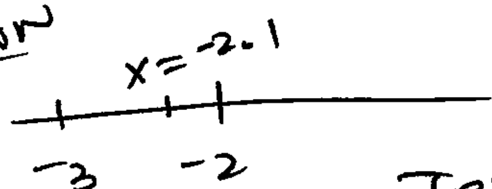
Therefore by Sq. Thm

$$\lim_{x \to 2} f(x) = 12$$

6. Find the infinite limit.

a) $\lim\limits_{x \to (-2)^-} \frac{3x}{x+2}$

SOLUTION

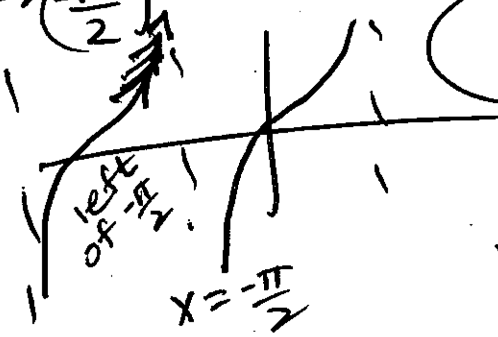


Test number
$x = -2.1$
$y = \frac{3(-2.1)}{-2.1+2}$
$= \frac{-6.3}{-.1} = 63$

Answer: $+\infty$

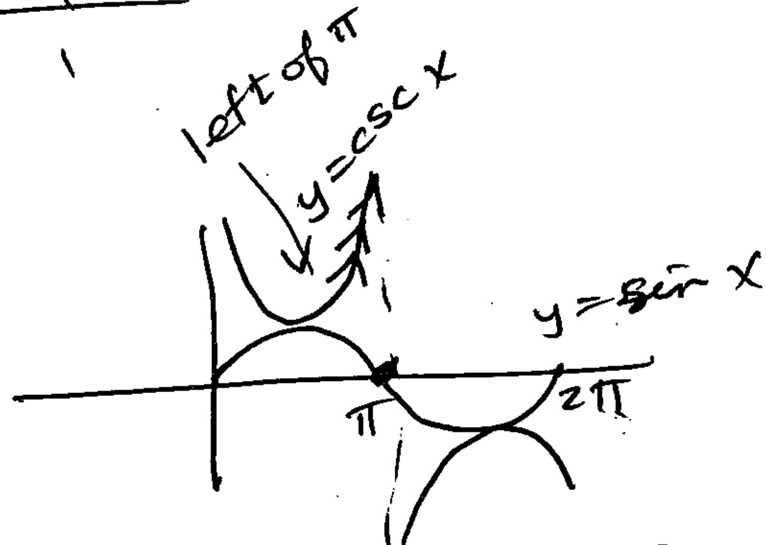
b) $\lim\limits_{x \to \left(-\frac{\pi}{2}\right)^-} \tan x$

Answer: ∞



c) $\lim\limits_{x \to \pi^-} \csc x$

$= \infty$



6

7. Find the limit.

a) $$\lim_{x\to\infty} \frac{3x^2-7x+2}{x^2+5}$$

$$= 3$$

b) $$\lim_{x\to\infty} \frac{3x+1}{x^2+5}$$

$$= 0$$

c) $$\lim_{x\to\infty} \frac{x^3+7x}{x^2+5}$$

$$= \lim_{x\to\infty} \frac{(x^3+7x)\ \frac{1}{x^2}}{(x^2+5)\ \frac{1}{x^2}} = \lim_{x\to\infty} \frac{\overset{\nearrow\infty}{x} + \overset{\nearrow 0}{7/x^2}}{1+\underset{\downarrow 0}{5/x^2}}$$

$$= \infty$$

8. Find the limit.

a) $$\lim_{x\to -1} \frac{x^2+3x+2}{x^3+1}$$

$$= \lim_{x\to -1} \frac{(x+2)\cancel{(x+1)}}{\cancel{(x+1)}(x^2-x+1)} = \frac{-1+2}{(-1)^2-(-1)+1} = \frac{1}{1+1+1} = \frac{1}{3}$$

b) $\lim_{x \to 4} \dfrac{\sqrt{x} - \cancel{4}2}{x - \cancel{4}4}$

$= \lim_{x \to 4} \left(\dfrac{\sqrt{x} - 2}{x-4}\right)\left(\dfrac{\sqrt{x}+2}{\sqrt{x}+2}\right)$

$= \lim_{x \to 4} \dfrac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x}+2)} = \lim_{x \to 4} \dfrac{1}{\sqrt{x}+2} = \dfrac{1}{\sqrt{4}+2} = \dfrac{1}{4}$

c) $\lim_{h \to 0} \dfrac{(2+h)^2 - 4}{h}$

$= \lim_{h \to 0} \dfrac{(4 + 4h + h^2) - 4}{h}$

$= \lim_{h \to 0} \dfrac{4h + h^2}{h} = \lim_{h \to 0} \dfrac{\cancel{h}(4+h)}{\cancel{h}} = 4 + 0 = 4.$

8