Math 180
Thurs Feb. 25

Practice Problems

Differentiate

① $y = \sqrt{x} + \frac{3}{\sqrt{x}}$

$y = x^{1/2} + 3x^{-1/2}$

$y' = \frac{1}{2}x^{-1/2} - \frac{3}{2}x^{-3/2}$

$y' = \frac{1}{2\sqrt{x}} - \frac{3}{2x\sqrt{x}}$

② $\cancel{y = x^2}$ $y = \frac{x^2+x+1}{\sqrt{x}}$

$y = (x^2+x+1)x^{-1/2}$

$y = x^{3/2} + x^{1/2} + x^{-1/2}$

$y' = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2}$

$y' = \frac{3}{2}\sqrt{x} + \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$

③ $y = x^2 \sin x$

$y' = (x^2)'(\sin x) + (x^2)(\sin x)'$

$y' = 2x\sin x + x^2\cos x$

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(4) $y = \sqrt{x}\tan x$

$y = x^{1/2}\tan x$

$y' = (x^{1/2})'(\tan x) + (x^{1/2})(\tan x)'$

$y' = \frac{1}{2}x^{-1/2}\tan x + x^{1/2}\sec^2 x$

$y' = \frac{1}{2\sqrt{x}}\tan x + \sqrt{x}\sec^2 x$

(5) $y = \frac{x^2+2}{x^2+5}$

$y' = \frac{(x^2+2)'(x^2+5) - (x^2+2)(x^2+5)'}{(x^2+5)^2}$

$y' = \frac{2x(x^2+5) - (x^2+2)(2x)}{(x^2+5)^2}$

$y' = \frac{\cancel{2x^3} + 10x - \cancel{2x^3} - 4x}{(x^2+5)^2}$

$y' = \frac{6x}{(x^2+5)^2}$

⑥ $$y = \frac{1+\sin x}{\cos x}$$

$$y' = \frac{(1+\sin x)'(\cos x) - (1+\sin x)(\cos x)'}{(\cos x)^2}$$

$$y' = \frac{(\cos x)(\cos x) - (1+\sin x)(-\sin x)}{\cos^2 x}$$

$$y' = \frac{\cos^2 x + \sin x + \sin^2 x}{\cos^2 x}$$

$$y' = \frac{(\cos^2 x + \sin^2 x) + \sin x}{\cos^2 x}$$

$$y' = \frac{1+\sin x}{\cos^2 x}$$

⑦ $$y = \frac{e^x+1}{e^x-1}$$

$$y' = \frac{(e^x+1)'(e^x-1) - (e^x+1)(e^x-1)'}{(e^x-1)^2}$$

$$y' = \frac{e^x(e^x-1) - (e^x+1)(e^x)}{(e^x-1)^2}$$

$$y' = \frac{\cancel{e^{2x}} - e^x - \cancel{e^{2x}} - e^x}{(e^x-1)^2} = \frac{-2e^x}{(e^x-1)^2}$$

§3.3 Continued.

Example: Assume $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

Evaluate.

① $\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = \lim_{\theta \to 0} \frac{1}{\underbrace{\left(\frac{\sin\theta}{\theta}\right)}_{\to 1}} = \frac{1}{1} = 1$

② $\lim_{x \to 0} \frac{\sin(5x)}{x} = \lim_{x \to 0} 5 \cdot \underbrace{\left(\frac{\sin(5x)}{(5x)}\right)}_{\to 1} = 5 \cdot 1 = 5$

③ $\lim_{x \to 0} \frac{\tan(3x)}{x}$

$= \lim_{x \to 0} \frac{1}{x} \cdot \tan(3x)$

$= \lim_{x \to 0} \frac{1}{x} \cdot \frac{\sin(3x)}{\cos(3x)}$

$= \lim_{x \to 0} 3 \underbrace{\left(\frac{\sin(3x)}{(3x)}\right)}_{\to 1} \cdot \frac{1}{\cos(3x)} = 3(1)(1) = 3$

$\cos(3x) \to \cos(3 \cdot 0) = \cos(0) = 1$

④

§3.4 The Chain Rule

HW §3.4 #1-54, 65, 66

Example (Pre-calc): Let $f(x) = x^2$,
$g(x) = 3x+2$.

Find

a) $f(g(x)) = f(3x+2) = (3x+2)^2$

we write $= f \circ g(x)$

b) $g(f(x)) = g(x^2) = 3(x^2)+2$

$= g \circ f(x)$

Example: Find the derivative

(i) $y = (x^2+1)^5$ ← outside function $y = u^5$

↑ inside function $u = x^2+1$

The derivative of a composition of functions is the derivative of ~~the~~ the outside function multiplied by the derivative of the inside function.

$y' = 5(x^2+1)^4 \cdot 2x = 10x(x^2+1)^4$

↑ $\frac{d}{dx}(x^2+1)$

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$y = \sin u$
$u = x^2+1$

(ii) $y = \sin(x^2+1)$

$y' = \cos(x^2+1) \cdot 2x$

(iii) $y = \tan(x^2+1)$

$y' = \sec^2(x^2+1) \cdot 2x$

$y' = 2x\sec^2(x^2+1)$

$y = \tan u$

$\frac{dy}{du} = \sec^2 u$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= (\sec^2 u)(2x)$

$= \sec^2(x^2+1) \cdot (2x)$

$= 2x\sec^2(x^2+1)$

Chain Rule

$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

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(iv) $y = \sqrt{x^3+1}$

$y = (x^3+1)^{1/2}$

$y' = \frac{1}{2}(x^3+1)^{-1/2} \cdot \underbrace{3x^2}_{\text{deriv. of inside}}$

$$y' = \frac{3x^2}{\sqrt{x^3+1}}$$

(v) $y = e^{(5x)}$

$y' = 5e^{(5x)}$

(vi) $y = e^{(\sin x)}$

$y' = e^{\sin x} \cdot \cos x$

(vii) $y = x^2 \cdot e^{(7x)}$

product rule

$y' = (x^2)'(e^{7x}) + (x^2)(e^{7x})'$ ← chain rule

$y' = 2x e^{7x} + x^2 \cdot (7e^{7x})$

$y' = e^{7x}(2x + 7x^2)$

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FORMULA $\frac{d}{dx} a^x = a^x \ln a$

Example: Find y'

① $y = 2^x$

$y' = 2^x \ln 2$

② $y = 5^x$

$y' = 5^x \ln 5$

③ $y = 7^x$

$y' = 7^x \ln 7$

④ $y = 7^{(x^2+1)}$

$y' = 7^{(x^2+1)} \ln 7 \cdot 2x$

↑ deriv. inside function