§3.6 Derivatives of Log Functions

HW §3.6 #1-34, 37-48

FORMULA $\frac{d}{dx}(\ln x) = \frac{1}{x}$

Example: Differentiate.

a) $y = \ln(x^2+1)$

$y = \ln u$

$\frac{dy}{du} = \frac{1}{u}$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$y' = \frac{1}{(x^2+1)} \cdot 2x$

inside function

$y' = \frac{2x}{x^2+1}$

b) $y = \ln(\sin x)$

$y' = \frac{1}{(\sin x)} \cdot \cos x = \cot x$

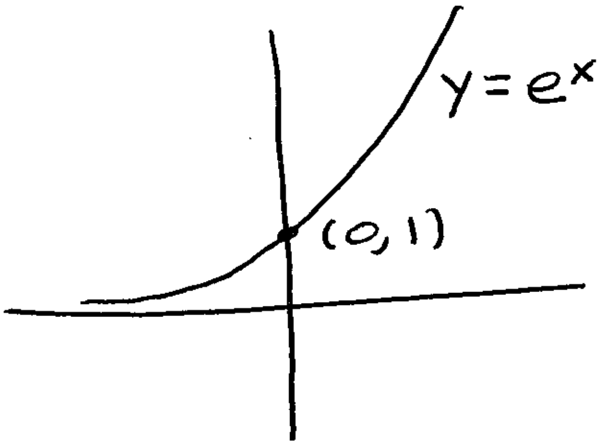
inside function

deriv. of sin x

c) $y = \ln(e^x + x)$

$y' = \frac{e^x+1}{(e^x+x)}$

Review of logarithmic functions

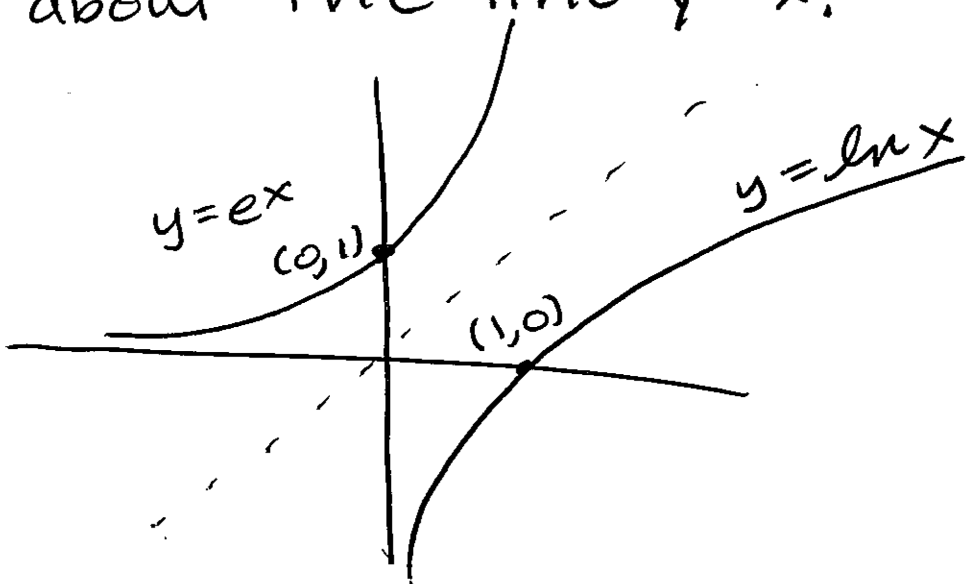


The inverse of $y=e^x$ is $y=\ln x$.

By definition, $y=\ln x$ is equivalent to $e^y=x$.

~~Note~~ By definition $\ln x=\log_e x$

The graph of $y=\ln x$ is found by reflecting the graph of $y=e^x$ about the line $y=x$.



LAWS OF LOGS

1. $\ln(AB) = \ln A + \ln B$
2. $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$
3. $\ln A^x = x \ln A$

Let's prove that $\frac{d}{dx} \ln x = \frac{1}{x}$.

PF Let $y = \ln x$

this is the same as $e^y = x$

Now take the deriv. of both sides with respect to x.

$$\frac{d}{dx} e^y = \frac{d}{dx} x$$

$$e^y \cdot y' = 1$$

$$y' = \frac{1}{e^y} \quad \text{but} \quad x = e^y$$

so $y' = \frac{1}{x}$ $\square$

FORMULA: $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

EXAMPLE: Differentiate.

a) $y = \ln \sqrt{x^2+1}$

$y = \ln (x^2+1)^{1/2}$

$y = \frac{1}{2} \ln (x^2+1)$

$y' = \frac{1}{\cancel{2}} \frac{1}{(x^2+1)} \cdot \cancel{2}x$

$y' = \frac{x}{x^2+1}$

b) $y = \ln (x+1)^3 \cdot (x+2)^5$

$y = \ln (x+1)^3 + \ln (x+2)^5$

$y = 3 \ln (x+1) + 5 \ln (x+2)$

$y' = 3\left(\frac{1}{x+1}\right) \cdot 1 + 5 \left(\frac{1}{x+2}\right) \cdot 1$

$y' = \frac{3}{x+1} + \frac{5}{x+2}$

Logarithmic Differention

Example: Use log. diff. to find y'

a) $$y = \frac{(x+3)^5 (x+1)^4}{\sqrt{x^2+5}}$$

We apply ln to both sides

$$\ln y = \ln \frac{(x+3)^5 (x+1)^4}{(x^2+5)^{1/2}}$$

$$\ln y = \ln (x+3)^5 + \ln (x+1)^4 - \ln (x^2+5)^{1/2}$$

$$\ln y = 5 \ln (x+3) + 4 \ln (x+1) - \frac{1}{2} \ln (x^2+5)$$

Now differentiate both sides using implicit diff.

$$\frac{1}{y} \cdot y' = 5\left(\frac{1}{x+3}\right) + 4\left(\frac{1}{x+1}\right) - \frac{1}{\cancel{2}}\left(\frac{1}{x^2+5}\right) \cdot \cancel{2}x$$

deriv of inside

Multiply through by y.

$$y' = y\left(\frac{5}{x+3} + \frac{4}{x+1} - \frac{x}{x^2+5}\right)$$

Substitute back for y.

$$y' = \frac{(x+3)^5 (x+1)^4}{\sqrt{x^2+5}}\left(\frac{5}{x+3} + \frac{4}{x+1} - \frac{x}{x^2+5}\right)$$

This is y.