Thurs, 4 March 2010

§ 3.6 Practice

Differentiate

① $y = \ln(\sec x)$

SOLUTION $y' = \frac{1}{\sec x} \cdot (\sec x)'$

$= \frac{1}{\sec x}(\sec x \tan x)$

$= \tan x$

② $y = \ln(x^5+3)$

SOLUTION $y' = \left(\frac{1}{x^5+3}\right)(x^5+3)'$

$= \frac{5x^4}{x^5+3}$

③ $y = \ln\sqrt{x^2+1}$

SOLUTION: First use LAWS OF LOGS to expand.

$y = \ln(x^2+1)^{1/2}$

$y = \frac{1}{2}\ln(x^2+1)$ Now find y'

$y' = \frac{1}{2}\left(\frac{1}{x^2+1}\right)(x^2+1)'$

$y' = \frac{1}{2}\left(\frac{1}{x^2+1}\right)(2x)$

$y' = \frac{x}{x^2+1}$

(4) $y = \ln \frac{(x^2+1)^4}{x^3\sqrt{x+5}}$

SOLUTION: First expand using Laws of Logs

$$y = \ln (x^2+1)^4 - \ln x^3 - \ln (x+5)^{1/2}$$

$$y' = 4 \ln (x^2+1) - 3 \ln x - \frac{1}{2} \ln (x+5)$$

$$y' = 4 \left(\frac{1}{x^2+1}\right)(x^2+1)' - 3\left(\frac{1}{x}\right) - \frac{1}{2}\left(\frac{1}{x+5}\right)$$

$$y' = 4 \left(\frac{1}{x^2+1}\right) 2x - \frac{3}{x} - \frac{1}{2(x+5)}$$

$$y' = \frac{8x}{x^2+1} - \frac{3}{x} - \frac{1}{2(x+5)}$$

§3.6 (continued)

Example: Use implicit differentiation to find y'

$$y = \frac{\sqrt{x^2+5}}{(x^5+2)^3}$$

• Apply ln to both sides

$$\ln y = \ln \frac{(x^2+5)^{1/2}}{(x^5+2)^3}$$

• Expand

$$\ln y = \ln (x^2+5)^{1/2} - \ln (x^5+2)^3$$

$$\ln y = \tfrac{1}{2}\ln (x^2+5) - 3\ln (x^5+2)$$

• Differentiate

$$\frac{1}{y}\cdot y' = \frac{1}{2}\left(\frac{1}{x^2+5}\right)\underbrace{(2x)}_{\text{deriv of } x^2+5} - 3\left(\frac{1}{x^5+2}\right)\cdot \underbrace{5x^4}_{\text{deriv } x^5+2}$$

• Multiply through by y.

$$y' = y\left[\frac{x}{x^2+5} - \frac{15x^4}{x^5+2}\right]$$

• Put back for y.

$$y' = \frac{\sqrt{x^2+5}}{(x^5+2)^3}\left[\frac{x}{x^2+5} - \frac{15x^4}{x^5+2}\right]$$

Example: Use Log. Diff. to find y':

$$y = x^{\sin x}$$

$$\ln y = \ln x^{\sin x}$$

$$\ln y = (\sin x)(\ln x)$$ ← We always get a product.

• Differentiate.

product rule

$$\frac{1}{y} y' = (\sin x)'(\ln x) + (\sin x)(\ln x)'$$

$$\frac{1}{y} y' = (\cos x)(\ln x) + (\sin x)\left(\frac{1}{x}\right)$$

$$y' = y\left[(\cos x)(\ln x) + \frac{\sin x}{x}\right]$$

$$y' = x^{\sin x}\left[(\cos x)(\ln x) + \frac{\sin x}{x}\right]$$

Homework Review

§3.4 # 13 Differentiate

$y = \cos(a^3 + x^3)$ chain rule

$y' = -\sin(a^3+x^3) \cdot (a^3+x^3)'$

$y' = -\sin(a^3+x^3) \cdot 3x^2$

$\boxed{\text{Note: } a^3 \text{ is a constant, so } (a^3)' = 0}$

$y' = -3x^2 \sin(a^3+x^3)$

§ 3.6 continued

Example: Use logarithmic differentiation to find y'.

$$y = \frac{(x^2+1)^5}{\sqrt{x^5+7}}$$

• Apply ln to both sides

$$\ln y = \ln \frac{(x^2+1)^5}{(x^5+7)^{1/2}}$$

• Expand

$$\ln y = 5\ln(x^2+1) - \frac{1}{2}\ln(x^5+7)$$

• Differentiate

$$\frac{1}{y}y' = 5\left(\frac{1}{x^2+1}\right)(x^2+1)' - \frac{1}{2}\left(\frac{1}{x^5+7}\right)(x^5+7)'$$

$$\frac{1}{y}y' = 5\left(\frac{1}{x^2+1}\right)(2x) - \frac{1}{2}\left(\frac{1}{x^5+7}\right)(5x^4)$$

• Multiply through by y.

$$y' = y\left[\frac{10x}{x^2+1} - \frac{5x^4}{2(x^5+7)}\right]$$

• Substitute back for y.

$$y' = \frac{(x^2+1)^5}{\sqrt{x^5+7}}\left[\frac{10x}{x^2+1} - \frac{5x^4}{2(x^5+7)}\right]$$

Example: Use log. diff. to find y': $y = x^{\cos x}$

SOLUTION

$$\ln y = \ln x^{\cos x}$$

$$\ln y = \cos x \ln x$$

$$\frac{1}{y} y' = (\cos x)'(\ln x) + (\cos x)(\ln x)'$$

$$\frac{1}{y} y' = (-\sin x)(\ln x) + (\cos x)\left(\frac{1}{x}\right)$$

$$y' = y\left((-\sin x)(\ln x) + \frac{\cos x}{x}\right)$$

$$y' = x^{\cos x}\left((-\sin x)(\ln x) + \frac{\cos x}{x}\right)$$

<u>Example</u> Use log diff. to find y'.

$$y = \frac{\sqrt{x^2+3}}{(x+5)^3}$$

• Apply ln to both sides

$$\ln y = \ln \frac{(x^2+3)^{1/2}}{(x+5)^3}$$

• Expand

$$\ln y = \ln (x^2+3)^{1/2} - \ln (x+5)^3$$

$$\ln y = \frac{1}{2}\ln(x^2+3) - 3\ln(x+5)$$

• Derivative

$$\frac{1}{y}y' = \frac{1}{\cancel{2}}\left(\frac{1}{x^2+3}\right)(\cancel{2}x) - 3\left(\frac{1}{x+5}\right)$$

$$y' = y\left[\frac{x}{x^2+3} - \frac{3}{x+5}\right]$$

$$y' = \frac{\sqrt{x^2+3}}{(x+5)^3}\left[\frac{x}{x^2+3} - \frac{3}{x+5}\right]$$

Example Find y' ~~using~~ using log. diff.

$$y = (\tan x)^x$$

$$\ln y = \ln (\tan x)^{x}$$

$$\ln y = x \cdot \ln(\tan x) \quad \text{product rule}$$

$$\frac{1}{y} y' = (x)'(\ln(\tan x)) + (x)(\ln(\tan x))'$$

$$\frac{1}{y} y' = 1 \cdot \ln(\tan x) + x \frac{1}{\tan x} \cdot \sec^2 x$$

$$y' = y\left[\ln(\tan x) + \frac{x\sec^2 x}{\tan x}\right]$$

$$y' = (\tan x)^x \left[\ln(\tan x) + \frac{x\sec^2 x}{\tan x}\right]$$