§3.4 Differentiate.

#37 $y = \cot^2(\sin\theta)$

$y = (\cot(\sin\theta))^2$

$y' = 2(\cot(\sin\theta)) \cdot (\cot(\sin\theta))'$

$y' = 2\cot(\sin\theta) \cdot \left(-\csc^2(\sin\theta)\right) \cdot (\sin\theta)'$

Let's start with something easier.

$y = \sin^3\theta$

use the chainrule

$y = (\sin\theta)^3$

$y' = 3(\sin\theta)^2 \cdot \cos\theta$

$y' = 3\sin^2\theta\cos\theta.$

$y' = -2\cot(\sin\theta)\csc^2(\sin\theta) \cdot \cos\theta$

§3.4 #54 Find the equation of the tangent line to $y = x^2 e^{-x}$ at $(1, 1/e)$.

SOLUTION

Find the slope.

$$y' = (x^2)'(e^{-x}) + (x^2)(e^{-x})'$$

$$y' = 2x e^{-x} + x^2(-e^{-x})$$

put in $x=1$, find m.

$$m = 2(1)e^{-1} + (1)^2(-e^{-1})$$

$$= \frac{2}{e} - \frac{1}{e} = \frac{1}{e}$$

$$x_1 = 1, \quad y_1 = 1/e, \quad m = \frac{1}{e}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{e} = \frac{1}{e}(x-1)$$

$$y - \cancel{\frac{1}{e}} = \frac{x}{e} - \cancel{\frac{1}{e}}$$

$$y = \frac{x}{e}$$

Follow-up. For which values of x is the tangent line horizontal?

Answer: when $y' = 0$.

$$2xe^{-x} - x^2e^{-x} = 0$$

$$e^{-x}(2x - x^2) = 0$$

$$e^{-x}\, x(2 - x) = 0$$

e^{-x} is never 0. $\quad x = 0, \quad x = 2$

#2

§3.4 # 53 $\quad y = \sin(\sin x), \quad (\pi, 0)$

Find the equation of the tangent line to the curve at the given point.

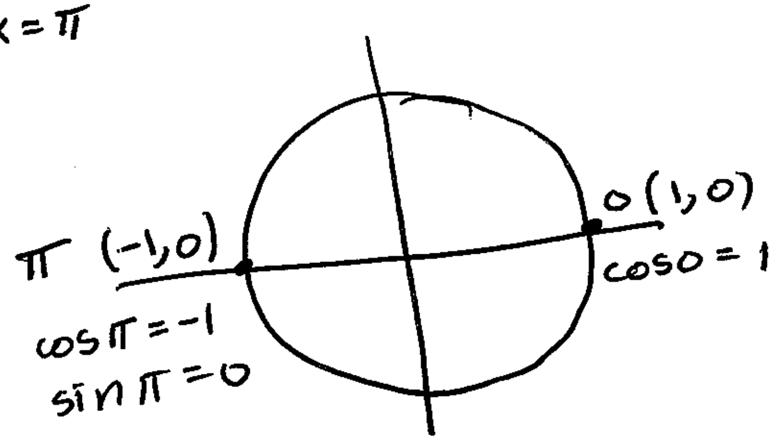
SOLUTION Find the slope.

$$y' = \cos(\sin x) \cdot \underbrace{\cos x}_{\text{deriv. of inside}}$$

$$m = \underbrace{y'(\pi)}_{\text{evaluate at } x=\pi} = \cos(\sin \pi) \cdot \cos \pi$$

$$= \cos(0) \cdot (-1)$$

$$= (1)(-1) = -1$$



$m = -1$

$x_1 = \pi$

$y_1 = 0$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1(x - \pi)$$

$$y = -x + \pi$$

§3.5 Find $\frac{dy}{dx}$ using implicit diff.

#15 $e^{x/y} = x - y$

$$\left(e^{x/y}\right) \cdot \left(\frac{x}{y}\right)' = 1 - y'$$

$$e^{x/y}\left(\frac{y - xy'}{y^2}\right) = 1 - y'$$

Aside:

$$\left(\frac{x}{y}\right)' = \frac{(x)'(y) - (x)(y)'}{(y)^2} = \frac{1 \cdot y - xy'}{y^2} = \frac{y - xy'}{y^2}$$

Solve for y'.

$$e^{x/y}\left(\frac{y - xy'}{y^2}\right) \cdot y^2 = (1 - y')y^2$$

$$e^{x/y}(y - xy') = y^2 - y'y^2$$

$$ye^{x/y} - xy'e^{x/y} = y^2 - y'y^2$$

$$-xy'e^{x/y} + y'y^2 = y^2 - ye^{x/y}$$

$$y'(-xe^{x/y} + y^2) = y^2 - ye^{x/y}$$

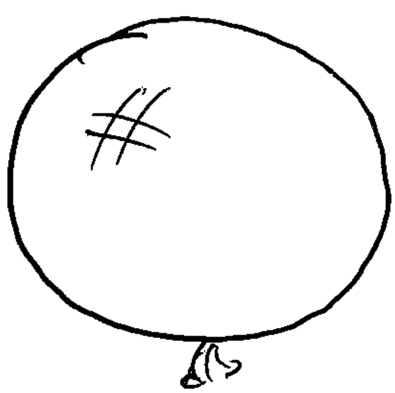
$$y' = \frac{y^2 - ye^{x/y}}{y^2 - xe^{x/y}}$$

§3.9 Related Rates

HW §3.9 # 1–19 odd

EXAMPLE A spherical balloon has pumped into it at a rate of 100 cm³/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

FIGURE



FACTS

$$\frac{d}{dt}V = 100\,\text{cm}^3/\text{s}$$

FORMULAS

$$V = \frac{4}{3}\pi r^3$$

Find $\frac{dr}{dt}$ when $r = 25$

Derivative

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \frac{dr}{dt}$$

FILL-IN: $r = 25, \quad \frac{dV}{dt} = 100$

$$100 = \left(4\pi (25)^2\right) \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{100}{4\pi \cdot (25)^2}$$

$$= \frac{10\cancel{0}}{\cancel{4}\pi \cdot \cancel{25} \cdot 25}$$

$$\boxed{\frac{dr}{dt} = \frac{1}{25\pi}} \approx 0.0127 \text{ cm/s}$$

Extra

Note:

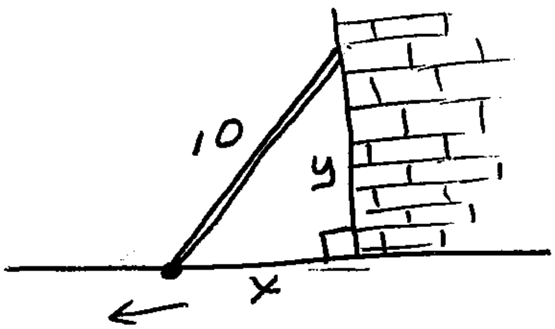
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100 = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{100}{4\pi r^2} = \frac{25}{\pi r^2}$$

EXAMPLE 2: A ladder 10 ft long rests against against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

SOLUTION



FORMULA

$$x^2+y^2=10^2$$

FACTS

$$\frac{dx}{dt}=1$$

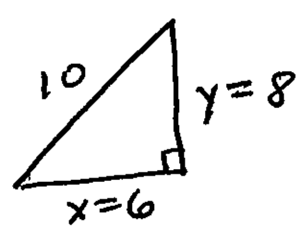
Find $\frac{dy}{dt}$ when $x=6$

Derivative:

$$x^2+y^2=100$$

$$\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=0$$

Snapshot when $x=6$



$$6^2+y^2=10^2$$

$$36+y^2=100$$

$$y^2=64$$

$$y=8$$

FILL-IN $x=6$, $\frac{dx}{dt}=1$, $y=?$ $y=8$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

$$6(1)+8\frac{dy}{dt}=0$$

$$8\frac{dy}{dt}=-6$$

$$\frac{dy}{dt}=\frac{-6}{8}=-\frac{3}{4}\text{ ft/s}$$

$\boxed{\text{The top slides down at } \frac{3}{4} \text{ ft/s}}$

More homework review

§ 3.5 # 25

Use implicit differentiation to find the equation of the tangent line to

$x^2 + xy + y^2 = 3$ at the point $(1,1)$

(ellipse)

<u>SOLUTION</u> Find y'.

$$2x + \underbrace{\left[(x)'(y) + (x)(y)'\right]}_{\text{productrule}} + 2y\,y' = 0$$

$$2x + 1 \cdot y + x\,y' + 2y\,y' = 0$$

put in $x = 1$, $y = 1$, $m = y'$

$$2(1) + (1) + (1)m + 2(1)m = 0$$

~~$3 = 3m$~~

$$3 + 3m = 0$$
$$3m = -3$$
$$m = -1$$

$x_1 = 1$, $y_1 = 1$, $m = -1$

$$y - y_1 = m(x - x_1)$$
$$y - 1 = -1(x - 1)$$
$$y = -1(x-1) + 1$$
$$y = -x + 1 + 1$$
$$y = -x + 2$$