Math 180
Tues 3/09/2010

Quiz Thurs

§3.5, 3.6

Homework Review

§3.5 Find $\frac{dy}{dx}$ by implicit differentation.

#7) $x^2 + xy - y^2 = 4$

$$2x + \left(\overbrace{(x)'(y) + (x)(y)'}^{\text{product rule}}\right) - 2y\,y' = 0$$

$$2x + (1)y + xy' - 2y\,y' = 0$$

$$xy' - 2y\,y' = -2x - y$$

$$y'(x - 2y) = -2x - y$$

$$y' = \frac{-2x - y}{x - 2y}$$

#

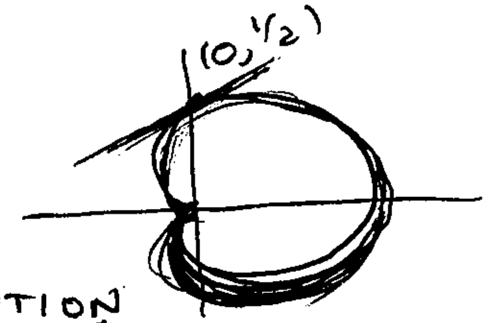
§ 3.5 # ~~28~~ 27 Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

under 6:30 / 8 min for 2000m

weight 190 lb

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2$$

at $(0, 1/2)$. Cardioid



SOLUTION

Find $\frac{dy}{dx}$.

chain rule   deriv. of inside

$$2x + 2yy' = 2(2x^2+2y^2-x) \cdot (4x + 4yy' - 1)$$

Now put in $x=0$, $y=1/2$, $m = y'$

$$2(0) + 2(\tfrac{1}{2})m = 2(2\cdot 0^2 + 2(\tfrac{1}{2})^2 - 0)\cdot(4(0) + 4(\tfrac{1}{2})m \cdot$$

$$m = 2(\tfrac{1}{2})\cdot(2m-1)$$

$$m = 2m - 1$$

$$0 = m - 1$$

$$m = 1$$

$$x_1 = 0, \quad y_1 = \tfrac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - \tfrac{1}{2} = 1(x - 0)$$

$$y - \tfrac{1}{2} = x, \qquad y = x + \tfrac{1}{2}$$

§3.6 #19 Find the derivative.

$$y = \ln(e^{-x} + xe^{-x})$$

$$y = \ln\left[e^{-x} \cdot (1+x)\right]$$

$$y = \ln(e^{-x}) + \ln(1+x)$$

$$y = -x \cdot \underbrace{\ln e}_{=1} + \ln(1+x)$$

$$y = -x + \ln(1+x)$$

$$\boxed{y' = -1 + \frac{1}{1+x}}$$

Use logarithmic differentiation to find the derivative of the function.

§3.6 #39 $\quad y = \dfrac{\sin^2 x \tan^4 x}{(x^2+1)^2}$

$$\ln y = \ln \frac{\sin^2 x \tan^4 x}{(x^2+1)^2}$$

$$\ln y = \ln \sin^2 x + \ln \tan^4 x - \ln (x^2+1)^2$$

$$\ln y = 2\ln(\sin x) + 4\ln(\tan x) - 2\ln(x^2+1)$$

Take deriv.

$$\frac{1}{y} y' = 2\left(\frac{1}{\sin x}\right)\cos x + \left(\frac{4}{\tan x}\right)\cdot \sec^2 x - 2\left(\frac{1}{x^2+1}\right)(2$$

$$y' = y\left[2\cot x + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right]$$

$$y' = \frac{\sin^2 \tan^4 x}{(x^2+1)^2}\left[2\cot x + 4\frac{\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right]$$

§3.6 #15 Differentiate

$$f(u) = \frac{\ln u}{1 + \ln(2u)}$$

<u>SOLUTION</u>

$$f'(u) = \frac{(\ln u)'(1 + \ln 2u) - (\ln u)(1 + \ln(2u))'}{(1 + \ln(2u))^2}$$

$$= \frac{\left(\frac{1}{u}\right)(1 + \ln 2u) - (\ln u)\left(\frac{1}{\cancel{2}u} \cdot \cancel{2}\right)}{(1 + \ln(2u))^2}$$

$$= \frac{\frac{1}{u}(1 + \ln 2u - \ln u)}{(1 + \ln(2u))^2}$$

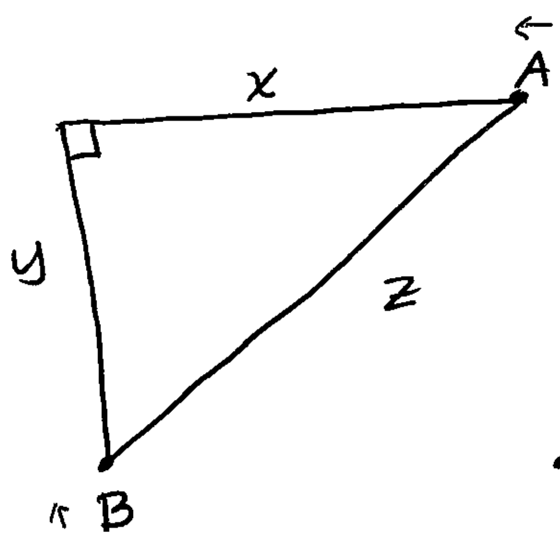
$$= \frac{(1 + \ln 2 + \cancel{\ln u} - \cancel{\ln u})}{u(1 + \ln(2u))^2}$$

$$= \frac{1 + \ln 2}{u(1 + \ln(2u))^2}$$

§3.9 notes continued.

Skip example 3.

EXAMPLE 4 . Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the intersection of the same road. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?



- Find: $\frac{dz}{dt}$ when $x = 0.3$, $y = 0.4$

- FORMULA $x^2 + y^2 = z^2$

FACTS: $\frac{dx}{dt} = -50$, $\frac{dy}{dt} = -60$

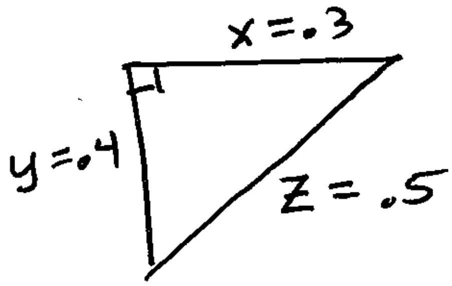
DERIVATIVE: $\cancel{2}x \cdot x' + \cancel{2}y \cdot y' = \cancel{2}z\, z'$

FILL-IN $x = .3,\ x' = -50,\ y = 0.4,\ y' = -60$

$z = ?$

$z = .5$

Snapshot



$(.3)^2 + (.4)^2 = z^2$

$z = .5$

$x x' + y y' = z z'$

$(.3)(-50) + (.4)(-60) = .5 z'$

$-15 + {}^{-}24 = \frac{1}{2} z'$

$2(-15 + {}^{-}24) = z'$

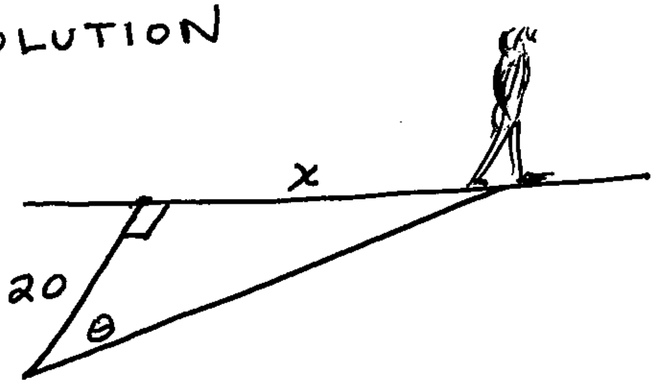
$z' = (-39)2$

$= -78$

Answer: 78 mi/h.

EXAMPLE 5 A man walks along a straight path at a speed of 4 ft/s. A search light is located on the ground 20 ft from the path and is kept focused on the man. At what rate the search light rotating when the man is 15 ft from the point on the path closest to the searchlight?

SOLUTION



FACTS: $\frac{dx}{dt} = 4$ ft/s

FIND: $\frac{d\theta}{dt}$ when $x = 15$

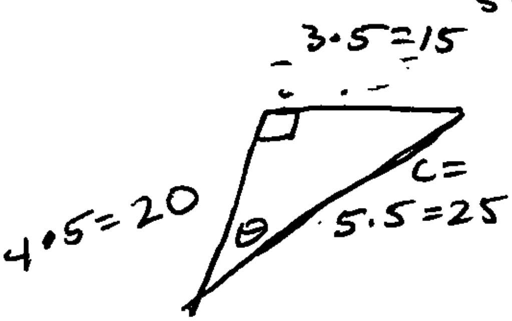
FORMULA:

$$\tan\theta = \frac{x}{20}$$

DERIVATIVE

$$(\sec^2\theta)\frac{d\theta}{dt} = \frac{1}{20}\frac{dx}{dt}$$

FILL-IN when $x = 15$, we need to find $\sec^2\theta$.



$$\cos\theta = \frac{adj}{hyp} = \frac{20}{25} = \frac{4}{5}$$

$$\sec\theta = \frac{5}{4}$$

$$\sec^2\theta = \frac{25}{16}$$

$$(\sec^2\theta)\left(\frac{d\theta}{dt}\right) = \frac{1}{20}\left(\frac{dx}{dt}\right)$$

$$\frac{25}{16}\left(\frac{d\theta}{dt}\right) = \frac{1}{20}(4)$$

$$\frac{d\theta}{dt} = \frac{16}{25}\cdot\frac{1}{\cancel{20}_{5}}\cdot\cancel{4}$$

$$\frac{d\theta}{dt} = \frac{16}{125} \text{ rad/s}$$

$$= 0.128 \text{ rad/s}$$

§3.5 #8 Find $\frac{dy}{dx}$ using implicit differentiation.

$$2x^3 + x^2y - xy^3 = 2$$

SOLUTION

$$6x^2 + \overbrace{\left((x^2)'(y) + (x^2)(y)'\right)}^{\text{product rule}} - \overbrace{\left((x)'(y^3) + (x)(y^3)'\right)}^{\text{productrule}} = 0$$

$$6x^2 + (2xy + x^2y') - (1 \cdot y^3 + x \cdot 3y^2y') = 0$$

$$6x^2 + 2xy + x^2y' - y^3 - 3xy^2y' = 0$$

$$x^2y' - 3xy^2y' = -6x^2 - 2xy + y^3$$

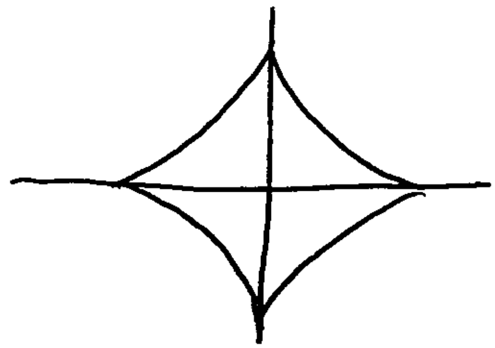
$$y'(x^2 - 3xy^2) = -6x^2 - 2xy + y^3$$

$$y' = \frac{-6x^2 - 2xy + y^3}{x^2 - 3xy^2}$$

§3.5 # 28 $\quad x^{2/3} + y^{2/3} = 4$

Find the tangent line at the point $(-3\sqrt{3}, 1)$

(astroid)



<u>SOLUTION</u> Find the slope

$$x^{2/3} + y^{2/3} = 4$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$

$$\frac{2}{3}y^{-1/3}y' = \frac{-2}{3}x^{-1/3}$$

$$y' = \frac{-2/3\, x^{-1/3}}{2/3\, y^{-1/3}}$$

$$y' = -\frac{x^{-1/3}}{y^{-1/3}}$$

$$y' = -\frac{y^{1/3}}{x^{1/3}}$$

put in $x = -3\sqrt{3} = -3 \cdot 3^{1/2}$
$= -3^{3/2}$

$y = 1$

$$y' = -\frac{(1)^{1/3}}{(-3^{3/2})^{1/3}}$$

$$= \frac{-1}{-3^{(3/2 \cdot \frac{1}{3})}} = \frac{1}{3^{1/2}}$$

$$m = \frac{1}{\sqrt{3}}$$

$x_1 = -3\sqrt{3}, \quad y_1 = 1, \quad m = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{\sqrt{3}}(x - (-3\sqrt{3}))$$

$$y - 1 = \frac{1}{\sqrt{3}}x + 3$$

$$y = \frac{1}{\sqrt{3}}x + 3 + 1$$

$$y = \frac{\sqrt{3}}{3}x + 4$$

§3.6 #13 differentiate.

$$g(x) = \ln\left(x \cdot \sqrt{x^2-1}\right)$$

SOLUTION

$$g(x) = \ln x + \ln (x^2-1)^{1/2}$$

$$= \ln x + \ln \left((x-1)(x+1)\right)^{1/2}$$

$$= \ln x + \ln (x-1)^{1/2} (x+1)^{1/2}$$

$$= \ln x + \ln (x-1)^{1/2} + \ln (x+1)^{1/2}$$

$$g(x) = \ln x + \tfrac{1}{2} \ln (x-1) + \tfrac{1}{2} \ln (x+1)$$

$$g'(x) = \frac{1}{x} + \frac{1}{2}\left(\frac{1}{x-1}\right) + \frac{1}{2}\left(\frac{1}{x+1}\right)$$

§3.6 # 42 Use logarithmic differentiation to find y'.

$$y = x^{\cos x}$$

SOLUTION

$$\ln y = \ln x^{(\cos x)}$$

expand

$$\ln y = (\cos x)(\ln x)$$

$$\frac{1}{y} y' = (\cos x)'(\ln x) + (\cos x)(\ln x)' \quad \text{differentiate}$$

$$\frac{1}{y} \cdot y' = (-\sin x)(\ln x) + (\cos x)\left(\frac{1}{x}\right)$$

$$y' = y\left[(-\sin x)(\ln x) + \frac{\cos x}{x}\right]$$

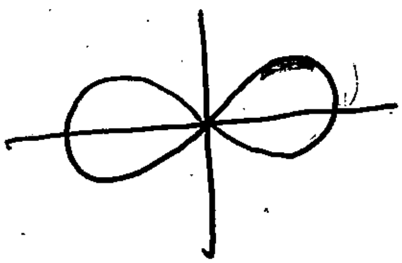
$$y' = x^{\cos x}\left[(-\sin x)(\ln x) + \frac{\cos x}{x}\right]$$

§3.5 #29 Find the equation of tangent line ~~to~~ at the given point.

$$2(x^2+y^2)^2 = 25(x^2-y^2)$$

$(3, 1)$

lemniscate



SOLUTION

Find the slope.

chain rule

$$2 \cdot 2(x^2+y^2) \cdot (2x+2yy')$$

↑ deriv. inside

$$= 25(2x-2yy')$$

put in $x=3$, $y=1$, $m=y'$

$$4(3^2+1^2)\cdot(2(3)+2(1)m) = 25(2\cdot 3 - 2\cdot 1m)$$

$$4(10)(6+2m) = 150 - 50m$$

$$240 + 80m = 150 - 50m$$

$$130m = \cancel{-9} \; 150 - 240$$

$$130m = -90$$

$$m = \frac{-9}{13}$$

$x_1 = 3, y_1 = 1$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{-9}{13}(x-3)$$

$$y - 1 = \frac{-9}{13}x + \frac{27}{13}$$

$$y = \frac{-9}{13}x + \frac{27}{13} + \frac{13}{13}$$

$$\boxed{y = \frac{-9}{13}x + \frac{40}{13}}$$

§3.5 Find dy/dx by implicit differentiation.

#9) $x^4 \cdot (x+y) = y^2(3x-y)$

product rule

$(x^4)'(x+y) + (x^4)(x+y)' = (y^2)'(3x-y) + (y^2)(3x-y)'$

$4x^3(x+y) + (x^4)(1+y') = 2y \cdot y'(3x-y) + y^2(3-y')$

$\cancel{4x^3(x+y) + x}$

$4x^4 + 4x^3y + x^4 + x^4y' = 6xyy' - 2y^2y' + 3y^2 - y^2y'$

$x^4y' - 6xyy' + 2y^2y' + y^2y' = -4x^4 - 4x^3y - x^4 + 3y^2$

$y'(x^4 - 6xy + 2y^2 + y^2) = -5x^4 - 4x^3y + 3y^2$

$$y' = \frac{-5x^4 - 4x^3y + 3y^2}{x^4 - 6xy + 3y^2}$$

Quiz Thurs.
§ 3.5, 3.6

§3.6 #9 Differentiate

$$y = (\sin x) \cdot \ln(5x)$$

product rule

SOLUTION $y' = (\sin x)'(\ln 5x) + (\sin x)(\ln 5x)'$

$$y' = (\cos x)(\ln 5x) + (\sin x)\left(\frac{1}{\cancel{5}x} \cdot \cancel{5}\right)$$

$$y' = (\cos x)(\ln 5x) + \frac{\sin x}{x}$$

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§3.6 #45 Find the derivative using logarithmic differentiation.

$$y = (\cos x)^x$$

SOLUTION

$$\ln y = \ln (\cos x)^x$$

$$\ln y = x \cdot \ln(\cos x)$$

$$\frac{1}{y} y' = (x)'(\ln(\cos x)) + (x)(\ln(\cos x))'$$

$$\frac{1}{y} y' = 1 \cdot \ln(\cos x) + x\left(\frac{1}{\cos x}\right)(-\sin x)$$

$$y' = y\left[\ln(\cos x) - x \tan x\right]$$

= [illegible]

$$y' = (\cos x)^x\left[\ln(\cos x) - x \tan x\right]$$