Thurs. ~~3/11~~

Chapter 3 Test.

The Chapter 3 Test will be Thursday, March 18, 2010

# §3.11 Hyperbolic Functions

HW §3.11 #30–47

**Definition**

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

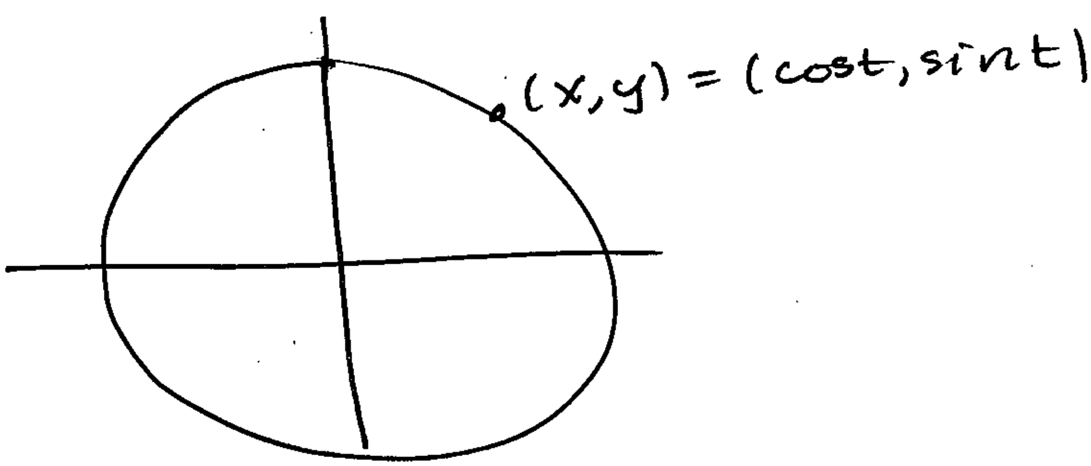
$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$

Identity: $\cosh^2 x - \sinh^2 x = 1$

If we let $x = \cos t$, $y = \sin t$, then

$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

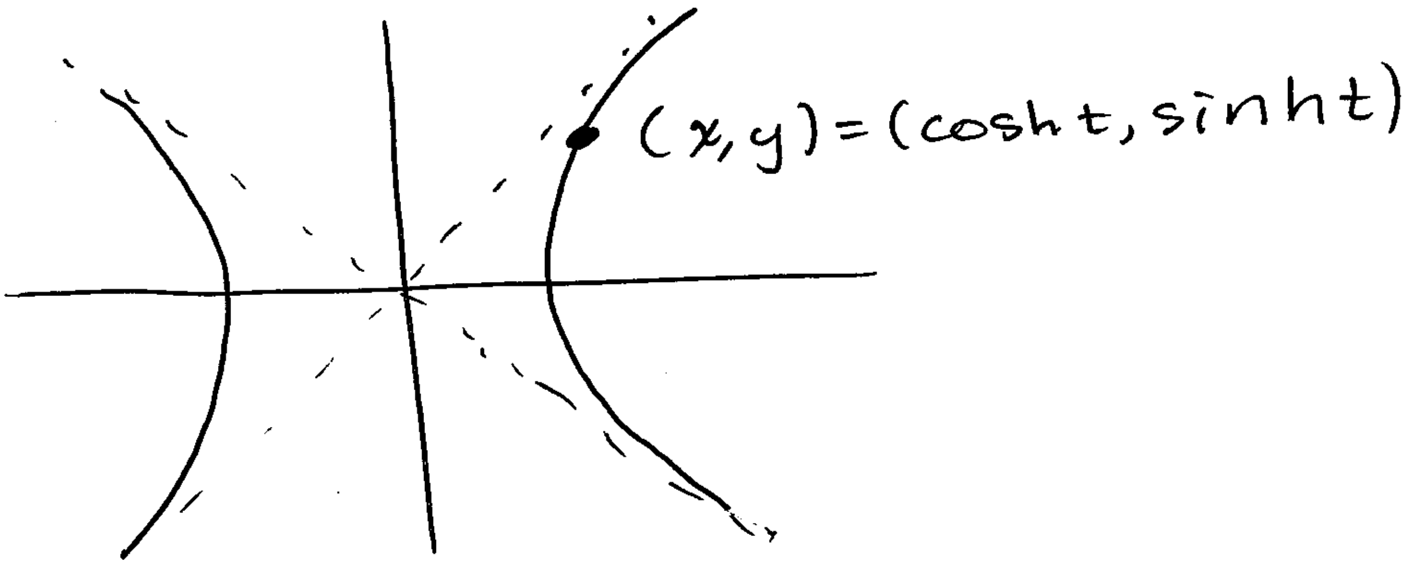
$x^2 + y^2 = 1$ is a unit circle



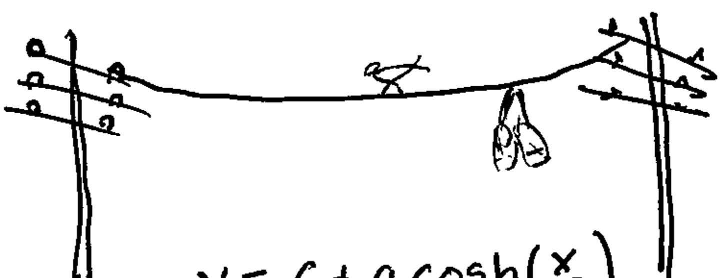
If we let $x = \cosh t$ and $y = \sinh t$
we get

$$x^2 - y^2 = \cosh^2 t - \sinh^2 t = 1$$

$x^2 - y^2 = 1$ is a hyperbola.




The most famous application is the use of hyperbolic cosine to describe the shape of a hanging wire.



$$y = c + a\cosh\left(\frac{x}{a}\right)$$

catenary

# Derivative Formulas

$\frac{d}{dx}(\sinh x) = \cosh x$ $\quad$ $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth$

$\frac{d}{dx}(\cosh x) = \sinh x$ $\quad$ $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$ $\quad$ $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Proof that $\frac{d}{dx}(\cosh x) = \sinh x$

Pf $\quad \frac{d}{dx}(\cosh x) = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right)$

$$= \frac{e^x - e^{-x}}{2} = \sinh x$$

## Inverse Hyperbolic Functions

$\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ $\quad x \in \mathbb{R}$

$\cosh^{-1} x = \ln(x + \sqrt{x^2-1})$ $\quad x \geq 1$

$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$ $\quad -1 < x < 1$

# Derivatives of Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{d}{dx}(\operatorname{csch}^{-1}x) = \frac{-1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2} \qquad \frac{d}{dx}(\coth^{-1}x) = \frac{1}{1-x^2}$$

<u>EXAMPLE</u>: Differentiate.

① $y = x\tanh^{-1}x + \ln\sqrt{1-x^2}$

$$y = x\tanh^{-1}x + \ln(1-x^2)^{1/2}$$

$$y = x\tanh^{-1}x + \frac{1}{2}\ln(1-x^2)$$

$$y' = (x)'(\tanh^{-1}x) + (x)(\tanh^{-1}x)' + \frac{1}{\cancel{2}}\left(\frac{1}{1-x^2}\right)(-\cancel{2}x)$$

$$y' = \tanh^{-1}x + x\left(\frac{1}{1-x^2}\right) + \left(\frac{-x}{1-x^2}\right)$$

$$y' = \tanh^{-1}x$$

(2) $y = \tanh^{-1}\sqrt{x}$

$$y = \tanh^{-1}(x^{1/2})$$

$$y' = \left(\frac{1}{1-(x^{1/2})^2}\right)\frac{d}{dx}(x^{1/2})$$

$$= \left(\frac{1}{1-x}\right)\frac{1}{2}x^{-1/2}$$

$$= \frac{1}{(1-x)}\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2(1-x)\sqrt{x}}$$

Let's do a homework ~~fr~~ problem.

§ 3.11 # 33

$$h(x) = \ln(\cosh x)$$

Find $h'(x)$.

$$h'(x) = \left(\frac{1}{\cosh x}\right)(\cosh x)'$$

$$= \frac{\sinh x}{\cosh x} = \tanh x$$

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$$y = x^2 \cdot \sinh^{-1}(2x)$$

$$y' = (x^2)'(\sinh^{-1}(2x)) + (x^2)(\sinh^{-1}(2x))'$$

$$= 2x \sinh^{-1}(2x) + x^2 \frac{1}{\sqrt{1+(2x)^2}} \cdot 2$$

$$= \boxed{2x\sinh^{-1}(2x) + \frac{2x^2}{\sqrt{1+(2x)^2}}}$$

~~d~~ ~~fro~~ formula

$$(\sinh^{-1} x)' = \frac{1}{\sqrt{1+x^2}}$$

§3.5 #33. Find $y''$ by implicit differentiation.

$$9x^2 + y^2 = 9$$

<u>SOLUTION</u> First find $y'$

$$18x + 2yy' = 0$$

$$2yy' = -18x$$

$$y' = \frac{-18x}{2y}$$

$$y' = -\frac{9x}{y}$$

Differentiate again.

$$y'' = \frac{(-9x)'(y) - (-9x)(y)'}{(y^2)}$$

$$y'' = \frac{-9y + 9xy'}{y^2}$$

put back in $y' = \frac{-9x}{y}$

$$y'' = \left(\frac{-9y + 9x\left(-\frac{9x}{y}\right)}{y^2}\right)\frac{y}{y}$$

multiply num & den by l.c.d.

$$y'' = \frac{-9y^2 - 81x^2}{y^3}$$

$$y'' = \frac{-9(y^2+9x^2)}{y^3}$$

Note:
$9x^2+y^2=9$
from the original problem.

$$y'' = \frac{-9(9)}{y^3}$$

$$\boxed{y'' = \frac{-81}{y^3}}$$