Formulas

- $\frac{d}{dx}\sin x = \cos x$
- $\frac{d}{dx}\cos x = -\sin x$
- $\frac{d}{dx}\tan x = \sec^2 x$
- $\frac{d}{dx}\cot x = -\csc^2 x$
- $\frac{d}{dx}\sec x = \sec x \tan x$
- $\frac{d}{dx}\csc x = -\csc x \cot x$
- $\frac{d}{dx}\ln|x| = \frac{1}{x}$
- $\frac{d}{dx}\log_a|x| = \frac{1}{x \ln a}$
- $\frac{d}{dx}e^x = e^x$
- $\frac{d}{dx}a^x = a^x \ln a$
- $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
- $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$
- $\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$
- $\frac{d}{dx}\csc^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$
- $\sin^2\theta + \cos^2\theta = 1$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
- $\cot^2\theta + 1 = \csc^2\theta$
- $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$
- $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$

- $M_n = \Delta x\left[f(\overline{x}_1) + f(\overline{x}_2) + \ldots + f(\overline{x}_n)\right]$
- $T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)\right]$
- $S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$
- If $|f''(x)| \leq K$ for $a \leq x \leq b$, then $|E_T| \leq \frac{K(b-a)^3}{12n^2}$ and $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
- If $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$, then $|E_S| \leq \frac{K(b-a)^5}{180n^4}$

Math 185 Chapter 7 Practice Test

Evaluate the integral using integration by parts.

1. $\int x^2 \sin 2x \; dx$

2. $\int \sin(\ln x)\ dx$

Evaluate the trigonometric integral.

3. $\int \sin^6 x \cos^3 x \; dx$

4. $\int_0^{\pi/2} \cos^2 x \; dx$

5. $\int \tan^3 x \sec^5 x \; dx$

Evaluate the integral using a trigonometric substitution.

6. $\displaystyle\int \frac{x^5}{\sqrt{x^2+4}}\,dx$

7. Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

(a) $\dfrac{1}{x^3 + 2x^2 + x}$

(b) $\dfrac{x - 1}{x^3 + x}$

(c) $\dfrac{x^2}{(x^2 + x + 1)^2}$

8. Evaluate the integral.

(a) $\displaystyle\int \frac{1}{(x+4)(x-1)}\,dx$

(b) $\int \frac{x^2 - x + 8}{x^3 + 4x} \, dx$

9. For finding an approximation using the Simpson's Rule, with $n = 8$, for the integral $\int_0^2 e^x \, dx$, state the following.

   (a) a

   (b) b

   (c) Δx

   (d) $x_0, x_1, x_2, \ldots, x_n$

   (e) Write out an expression for S_8, without evaluating the expression.

10. How large do we have to choose n so that the approximation for T_n, M_n and S_n to the integral $\int_0^2 e^x \, dx$ is accurate to within 0.00001?

11. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\int_{-\infty}^{\infty} x^2 e^{-x^3}\, dx$

(b) $\int_0^3 \frac{1}{x\sqrt{x}}\, dx.$

12. Use the Comparison Theorem to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{2+e^{-x}}{x}\,dx$$

Math 185 Chapter 7 Practice Test Solution Key

Evaluate the integral using integration by parts.

1. $\int x^2 \sin 2x \; dx$

$$u = x^2 \qquad dv = \sin 2x \; dx$$
$$du = 2x \; dx \qquad v = -\tfrac{1}{2}\cos 2x$$

$$\begin{aligned}
\int u \; dv &= uv - \int v \; du \\
&= x^2 \cdot (-\frac{1}{2}\cos 2x) - \int (-\frac{1}{2}\cos 2x) \cdot 2x \; dx \\
&= -\frac{x^2}{2}\cos 2x + \int x \cos 2x \; dx
\end{aligned}$$

Evaluate $\int x \cos 2x \; dx$.

$$u = x \qquad dv = \cos 2x \; dx$$
$$du = dx \qquad v = \tfrac{1}{2}\sin 2x$$

$$\begin{aligned}
\int u \; dv &= uv - \int v \; du \\
&= x \cdot (\frac{1}{2}\sin 2x) - \int \frac{1}{2}\sin 2x \; dx \\
&= \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x
\end{aligned}$$

Therefore

$$\int x^2 \sin 2x \; dx = -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C \quad \square$$

2. $\int \sin(\ln x)\ dx$

$$\begin{array}{ll} u = \sin(\ln x) & dv = \ dx \\ du = \cos(\ln x) \cdot \frac{1}{x}\ dx & v = x \end{array}$$

$$\begin{aligned} \int u\ dv &= uv - \int v\ du \\ &= \sin(\ln x) \cdot x - \int x \cdot \cos(\ln x) \cdot \frac{1}{x}\ dx \\ &= x \sin(\ln x) - \int \cos(\ln x)\ dx \end{aligned}$$

Now we apply integration by parts once again to $\int \cos(\ln x)\ dx$

$$\begin{array}{ll} u = \cos(\ln x) & dv = \ dx \\ du = -\sin(\ln x) \cdot \frac{1}{x}\ dx & v = x \end{array}$$

$$\begin{aligned} \int u\ dv &= uv - \int v\ du \\ &= \cos(\ln x) \cdot x - \int x \cdot \left(-\sin(\ln x) \cdot \frac{1}{x} \right)\ dx \\ &= x \cos(\ln x) + \int \sin(\ln x)\ dx \end{aligned}$$

If we put this together, we get

$$\begin{aligned} \int \sin(\ln x)\ dx &= x \sin(\ln x) - \left(x \cos(\ln x) + \int \sin(\ln x)\ dx \right) \\ &= x \sin(\ln x) - x \cos(\ln x) - \int \sin(\ln x)\ dx \end{aligned}$$

We now solve for $\int \sin(\ln x)\ dx$.

$$2 \int \sin(\ln x)\ dx = x \sin(\ln x) - x \cos(\ln x)$$

which implies

$$\int \sin(\ln x)\ dx = \frac{1}{2} \left[x \sin(\ln x) - x \cos(\ln x) \right] \quad \square$$

Evaluate the trigonometric integral.

3. $\int \sin^6 x \cos^3 x \ dx$

We use the identity $\sin^2 x + \cos^2 x = 1$ and make the u substitution

$$\begin{aligned} u &= \sin x \\ du &= \cos x \ dx \end{aligned}$$

$$\begin{aligned} \int \sin^6 x \cos^3 x \ dx &= \int \sin^6 x \cos^2 x \cdot \cos x \ dx \\ &= \int \sin^6 x (1 - \sin^2 x) \cdot \cos x \ dx \\ &= \int u^6 (1 - u^2) \ du \\ &= \int (u^6 - u^8) \ du \\ &= \frac{u^7}{7} - \frac{u^9}{9} + C \\ &= \frac{\sin^7 x}{7} - \frac{\sin^9 x}{9} + C \quad \square \end{aligned}$$

4. $\int_0^{\pi/2} \cos^2 x \ dx$

We will use the identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$.

$$\begin{aligned} \int_0^{\pi/2} \cos^2 x \ dx &= \int_0^{\pi/2} \frac{1}{2}(1 + \cos 2x) \ dx \\ &= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/2} \\ &= \frac{1}{2}\left[\frac{\pi}{2} + \frac{1}{2}\sin\left(2 \cdot \frac{\pi}{2}\right)\right] - \frac{1}{2}\left[0 + \frac{1}{2}\sin(2 \cdot 0)\right] \\ &= \frac{\pi}{4} \quad \square \end{aligned}$$

5. $\int \tan^3 x \sec^5 x \; dx$

We use the identity $\tan^x +1 = \sec^2 x$ and make the u substitution:

$$\begin{aligned} u &= \sec x \\ du &= \sec x \tan x \; dx \end{aligned}$$

$$\begin{aligned} \int \tan^3 x \sec^5 x \; dx &= \int \tan^2 x \sec^4 x \cdot \sec x \tan x \; dx \\ &= \int (\sec^2 x - 1) \sec^4 x \cdot \sec x \tan x \; dx \\ &= \int (u^2 - 1) u^4 \; du \\ &= \int (u^6 - u^4) \; du \\ &= \frac{u^7}{7} - \frac{u^5}{5} + C \\ &= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C \quad \square \end{aligned}$$

Evaluate the integral using a trigonometric substitution.

6. $\displaystyle\int \frac{x^5}{\sqrt{x^2+4}}\ dx$

We make the substitution:

$$\begin{aligned} x &= 2\tan\theta \\ dx &= 2\sec^2\theta\ d\theta \end{aligned}$$

and, using the identity $\tan^2\theta + 1 = \sec^2\theta$,

$$\begin{aligned} \sqrt{x^2+4} &= \sqrt{(2\tan\theta)^2+4} \\ &= \sqrt{4\tan^2\theta+4} \\ &= \sqrt{4(\tan^2\theta+1)} \\ &= \sqrt{4\sec^2\theta} \\ &= 2\sec\theta \end{aligned}$$

Making the substitution we get

$$\begin{aligned} \int \frac{x^5}{\sqrt{x^2+4}}\ dx &= \int \frac{(2\tan\theta)^5}{2\sec\theta}\cdot 2\sec^2\theta\ d\theta \\ &= 32\int \tan^5\theta\sec\theta\ d\theta \end{aligned}$$

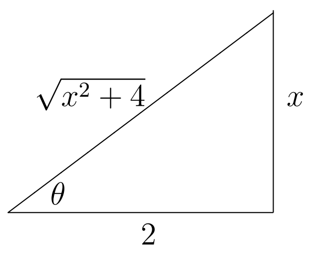
To solve this trig integral, we use the identity $\tan^x + 1 = \sec^2 x$ and make the u substitution:

$$\begin{aligned} u &= \sec x \\ du &= \sec x\tan x\ dx \end{aligned}$$

$$\begin{aligned} 32\int \tan^5\theta\sec\theta\ d\theta &= 32\int \tan^4\theta\sec\theta\tan\theta\ d\theta \\ &= 32\int (\tan^2\theta)^2\sec\theta\tan\theta\ d\theta \end{aligned}$$

$$
\begin{aligned}
&= 32\int (\sec^2\theta - 1)^2 \sec\theta\tan\theta \; d\theta \\
&= 32\int (u^2-1)^2 \; du \\
&= 32\int [(u^2)^2 - 2u^2 + 1] \; du \\
&= 32\int [u^4 - 2u^2 + 1] \; du \\
&= 32\left[\frac{u^5}{5} - 2\cdot\frac{u^3}{3} + u\right] + C \\
&= 32\left[\frac{\sec^5\theta}{5} - \frac{2}{3}\sec^3\theta + \sec\theta\right] + C
\end{aligned}
$$

Finally, we should write the integral in terms of the variable x. We know that $x = 2\tan\theta$, so we can draw a right triangle to solve for $\sec\theta$.



$$\tan\theta = \frac{x}{2}, \quad \cos\theta = \frac{2}{\sqrt{x^2+4}}, \quad \sec\theta = \frac{\sqrt{x^2+4}}{2}$$

Making this final substitution, we get

$$
\begin{aligned}
\int \frac{x^5}{\sqrt{x^2+4}}\, dx &= 32\left[\frac{1}{5}\left(\frac{\sqrt{x^2+4}}{2}\right)^5 - \frac{2}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3 + \frac{\sqrt{x^2+4}}{2}\right] + C \\
&= \left[\frac{1}{5}(x^2+4)^2 - \frac{8}{3}(x^2+4) + 16\right]\sqrt{x^2+4} + C
\end{aligned}
$$

I could go further on simplifying, but I am going to stop here. On the test, you can stop on the next to the last (the penultimate) line.

7. Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

(a) $$\frac{1}{x^3+2x^2+x} = \frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

(b) $$\frac{x-1}{x^3+x} = \frac{x-1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(c) $$\frac{x^2}{(x^2+x+1)^2} = \frac{Ax+B}{(x^2+x+1)} + \frac{Cx+D}{(x^2+x+1)^2}$$

8. Evaluate the integral.

(a) $\int \frac{1}{(x+4)(x-1)}\, dx$
First, we need to expand with partial fractions.

$$\frac{1}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$$

$$1 = A(x-1) + B(x+4)$$

Let $x = 1$. Then
$$1 = 5B \Rightarrow B = \frac{1}{5}$$

Let $x = -4$. Then
$$1 = -5A \Rightarrow A = -\frac{1}{5}$$

Therefore
$$\begin{aligned}\int \frac{1}{(x+4)(x-1)}\, dx &= \int -\frac{1}{5}\cdot\frac{1}{x+4} + \frac{1}{5}\cdot\frac{1}{x-1} \\ &= -\frac{1}{5}\ln|x+4| + \frac{1}{5}\ln|x-1| \quad \square\end{aligned}$$

(b) $\int \frac{x^2 - x + 8}{x^3 + 4x}\, dx$

$$\frac{x^2-x+6}{x^3+4x} = \frac{x^2-x+8}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$x^2 - x + 8 = A(x^2+4) + (Bx+C)x$$

Let $x = 0$. Then
$$8 = 4A \Rightarrow A = 2$$

Let $x = 1$. Using $A = 2$, we get

$$1^2 - 1 + 8 = 2 \cdot (1^2 + 4) + (B \cdot 1 + C) \cdot 1 \Rightarrow B + C = -2$$

Let $x = -1$. Using $A = 2$, we get

$$(-1)^2 - (-1) + 8 = 2 \cdot ((-1)^2 + 4) + (B \cdot (-1) + C) \cdot (-1) \Rightarrow B - C = 0$$

We can solve the system of equations below.

$$\begin{aligned} B + C &= -2 \\ B - C &= 0 \end{aligned}$$

$$\Rightarrow B = -1, \quad C = -1$$

Therefore

$$\begin{aligned} \int \frac{x^2 - x + 8}{x^3 + 4x}\, dx &= \int \frac{2}{x} - \frac{x + 1}{x^2 + 4}\, dx \\ &= \int \frac{2}{x}\, dx - \int \frac{x}{x^2 + 4}\, dx - \int \frac{1}{x^2 + 4}\, dx \end{aligned}$$

We will solve each of these integrals by a different method.

- $\displaystyle \int \frac{2}{x}\, dx = 2 \ln |x| + C$
- $\displaystyle \int \frac{x}{x^2 + 4}\, dx$

  Use the substitution method.

$$\begin{aligned} u &= x^2 + 4 \\ du &= 2x\ dx \\ \frac{1}{2} du &= x\ dx \end{aligned}$$

$$\begin{aligned}\int \frac{x}{x^2+4}\,dx &= \frac{1}{2}\int \frac{du}{u} \\ &= \frac{1}{2}\ln|u| + C \\ &= \frac{1}{2}\ln(x^2+4) + C\end{aligned}$$

- $\displaystyle\int \frac{1}{x^2+2^2}\,dx = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

  Here we use the formula

  $$\int \frac{1}{u^2+a^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

Finally, the answer is

$$\int \frac{x^2-x+8}{x^3+4x}\,dx = 2\ln|x| - \frac{1}{2}\ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C \quad \square$$

9. For finding an approximation using the Simpson's Rule, with $n = 8$, for the integral $\int_0^2 e^x \, dx$, state the following.

   (a) $a = 0$

   (b) $b = 2$

   (c) $\Delta x = \dfrac{b-a}{n} = \dfrac{2-0}{8} = \dfrac{1}{4} = 0.25$

   (d) $x_0, x_1, x_2, \ldots, x_n$

$$\begin{aligned} x_0 &= 0 \\ x_1 &= 0.25 \\ x_2 &= 0.5 \\ x_3 &= 0.75 \\ x_4 &= 1 \\ x_5 &= 1.25 \\ x_6 &= 1.5 \\ x_7 &= 1.75 \\ x_8 &= 2 \end{aligned}$$

   (e) Write out an expression for S_8, without evaluating the expression.

$$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

$$S_8 = \frac{0.25}{3}\left[f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + 2f(1) + 4f(1.25) + 2f(1.5) + 4f(1.75) + f(2)\right]$$

$$S_8 = \frac{0.25}{3}\left[e^{(0)} + 4e^{(0.25)} + 2e^{(0.5)} + 4e^{(0.75)} + 2e^{(1)} + 4e^{(1.25)} + 2e^{(1.5)} + 4e^{(1.75)} + e^{(2)}\right]$$

10. How large do we have to choose n so that the approximation for T_n, M_n and S_n to the integral $\int_0^2 e^x\,dx$ is accurate to within 0.00001?

If $|f''(x)| \leq K$ for $a \leq x \leq b$, then

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}$$

and

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}$$

If $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$, then

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

$f^{(k)}(x) = e^x$ for all positive integers k. Because $y = e^x$ is an increasing function, it takes its maximum value at $x = 2$. Therefore, we let $K = e^2$ for E_T, E_M and E_S.

In order to have $E_T =$ within 0.00001 of the given integral, we want

$$|E_T| \leq \frac{e^2(2-0)^3}{12n^2} = \frac{2e^2}{3n^2} < 0.00001$$

$$\Rightarrow \frac{2e^2}{3(0.00001)} < n^2$$

$$\Rightarrow \sqrt{\frac{2e^2}{3(0.00001)}} < n$$

$$\Rightarrow 701.9 < n$$

Choose $n = 702$.

In order to have $E_M =$ within 0.00001 of the given integral, we want

$$|E_M| \leq \frac{e^2(2-0)^3}{24n^2} = \frac{e^2}{3n^2} < 0.00001$$

$$\Rightarrow \frac{e^2}{3(0.00001)} < n^2$$

$$\Rightarrow \sqrt{\frac{e^2}{3(0.00001)}} < n$$

$$\Rightarrow 496.3 < n$$

Choose $n = 497$.

In order to have $E_S =$ within 0.00001 of the given integral, we want

$$|E_T| \leq \frac{e^2(2-0)^5}{180n^4} = \frac{8e^2}{45n^4} < 0.00001$$

$$\Rightarrow \frac{8e^2}{45(0.00001)} < n^4$$

$$\Rightarrow \sqrt[4]{\frac{8e^2}{45(0.00001)}} < n$$

$$\Rightarrow 19.03 < n$$

Choose $n = 20$. Note that for Simpson's Rule, n should be even.

11. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a) $\int_{-\infty}^{\infty} x^2 e^{-x^3}\ dx$

$$\begin{aligned}
\int_{-\infty}^{\infty} x^2 e^{-x^3}\ dx &= \int_{0}^{\infty} x^2 e^{-x^3}\ dx + \int_{-\infty}^{0}\ dx \\
&= \lim_{t\to\infty} \int_{0}^{t} x^2 e^{-x^3}\ dx + \lim_{t\to-\infty} \int_{t}^{0} x^2 e^{-x^3}\ dx
\end{aligned}$$

Make the following substitution:

$$\begin{aligned}
u &= -x^3 \\
du &= -3x^2\ dx \\
-\frac{1}{3}du &= x^2 dx
\end{aligned}$$

$$\begin{aligned}
\int x^2 e^{-x^3}\ dx &= -\frac{1}{3}\int e^u\ du \\
&= -\frac{1}{3}e^u \\
&= -\frac{1}{3}e^{-x^3}
\end{aligned}$$

$$\begin{aligned}
\int_{0}^{\infty} x^2 e^{-x^3}\ dx &= \lim_{t\to\infty} \int_{0}^{t} x^2 e^{-x^3}\ dx \\
&= \lim_{t\to\infty} \left[-\frac{1}{3}e^{-x^3}\right]_0^t \\
&= \lim_{t\to\infty} \left[-\frac{1}{3}e^{-t^3}\right] - \left[-\frac{1}{3}e^{-0^3}\right] \\
&= \left[-\frac{1}{3}\lim_{t\to\infty} e^{-t^3}\right] + \frac{1}{3} \\
&= \frac{1}{3}
\end{aligned}$$

$$
\begin{aligned}
\int_{-\infty}^{0} x^2 e^{-x^3}\,dx &= \lim_{t\to-\infty}\int_t^0 x^2 e^{-x^3}\,dx \\
&= \lim_{t\to-\infty}\left[-\frac{1}{3}e^{-x^3}\right]_t^0 \\
&= \lim_{t\to-\infty}\left[-\frac{1}{3}e^{-0^3}\right]-\left[-\frac{1}{3}e^{-t^3}\right] \\
&= -\frac{1}{3}+\left[\frac{1}{3}\lim_{t\to-\infty} e^{-t^3}\right] \\
&= \infty
\end{aligned}
$$

The integral is divergent.□

(b) $\int_0^3 \frac{1}{x\sqrt{x}}\,dx$.

$$
\begin{aligned}
\int_0^3 \frac{1}{x\sqrt{x}} &= \lim_{t\to0^+}\int_t^3 \frac{1}{x\sqrt{x}}\,dx \\
&= \lim_{t\to0^+}\int_t^3 x^{-3/2}\,dx \\
&= \lim_{t\to0^+}\left[\frac{x^{-1/2}}{-1/2}\right]_t^3 \\
&= -2\lim_{t\to0^+}\left[x^{-1/2}\right]_t^3 \\
&= -2\lim_{t\to0^+}\left[3^{-1/2}-t^{-1/2}\right] \\
&= -2\left[\frac{1}{\sqrt{3}}-\lim_{t\to0^+}\frac{1}{\sqrt{t}}\right]
\end{aligned}
$$

Because $\lim_{t\to0^+}\frac{1}{\sqrt{t}}=\infty$, the integral is divergent. □

12. Use the Comparison Theorem to determine whether the integral is convergent or divergent.

$$\int_1^\infty \frac{2+e^{-x}}{x}\,dx$$

$$0 \le \frac{1}{x} \le \frac{2+e^{-x}}{x}, \quad 1 \le x$$

The integral $\int_1^\infty \frac{1}{x}\,dx$ is divergent because it is of the form $\int_1^\infty \frac{1}{x^p}\,dx$ with $p = 1$. Therefore, by the Comparison Theorem, the integral $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$ is divergent.□