ID#:

Math 185 Chapter 7b Test

Show all of your work.

Evaluate the integral using integration by parts.

1. $\int \frac{\ln x}{\sqrt{x}} \; dx$

2. $\int e^x \cos 4x \; dx$

Evaluate the trigonometric integral.

3. $\int_0^{\pi/6} \sin^2 x \cos^3 x \, dx$

4. $\int \sec^4 x \tan^2 x \, dx$

5. Evaluate the integral using a trigonometric substitution.

$$\int \frac{x^2}{\sqrt{x^2 - 25}}\, dx$$

6. Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

(a) $\dfrac{1}{x^5 - 6x^4 + 9x^3}$

(b) $\dfrac{x - 7}{(x + 5)^4}$

(c) $\dfrac{x^2 + 5x + 1}{(x - 1)(x^2 + 4)^2(x - 3)^3}$

7. Evaluate the integral.

$$\int \frac{2x^3 - 3x^2 - 1}{x^2 - 3x + 2}\, dx$$

8. Evaluate the integral.

$$\int \frac{3x^2 - 8x + 36}{x^3 - 6x^2 + 18x}\, dx$$

9. How large do we have to choose n so that the approximation to the integral $\int_1^2 \frac{1}{x^2}\,dx$ is accurate to within 0.00001?

(a) For the error of the Midpoint Rule, E_M:

(b) For the error of Trapezoid Rule E_T:

(c) For the error of Simpson's Rule E_S:

Formulas

- $\int \sec u \, du = \ln|\sec u + \tan u| + C$
- $\int \sin^n u \, du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u \, du$
- $\int \cos^n u \, du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u \, du$
- $\int \sec^n u \, du = \frac{1}{n-1}\sec^{n-2} u \tan u + \frac{n-2}{n-1}\int \sec^{n-2} u \, du, \quad n \neq 1$
- $M_n = \Delta x\,[f(\overline{x}_1) + f(\overline{x}_2) + \ldots + f(\overline{x}_n)]$
- $T_n = \frac{\Delta x}{2}\,[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)]$
- $S_n = \frac{\Delta x}{3}\,[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$
- If $|f''(x)| \leq K$ for $a \leq x \leq b$, then $|E_T| \leq \frac{K(b-a)^3}{12n^2}$ and $|E_M| \leq \frac{K(b-a)^3}{24n^2}$
- If $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$, then $|E_S| \leq \frac{K(b-a)^5}{180n^4}$

Math 185 Chapter 7b Test

Show all of your work.

Evaluate the integral using integration by parts.

1. $\int \frac{\ln x}{\sqrt{x}}\, dx$

$u = \ln x \qquad dv = x^{-1/2}\, dx$

$du = \frac{1}{x}\, dx \qquad v = 2x^{1/2}$

$= 2x^{1/2} \ln x - \int 2x^{1/2} \cdot \frac{1}{x}\, dx$

$= 2\sqrt{x} \ln x - 2\int x^{-1/2}\, dx$

$= 2\sqrt{x} \ln x - 2 \cdot 2x^{1/2} + C$

$= 2\sqrt{x} \ln x - 4\sqrt{x} + C$

$I =$ 2. $\int e^x \cos 4x \quad dx$

$u = e^x \qquad dv = \cos 4x\, dx$

$du = e^x\, dx \qquad v = \frac{1}{4} \sin 4x$

$I = e^x \cdot \frac{1}{4} \sin 4x - \frac{1}{4} \int e^x \sin 4x\, dx$

$u = e^x \qquad dv = \sin 4x\, dx$

$du = e^x\, dx \qquad v = -\frac{1}{4} \cos 4x$

$= -\frac{1}{4} e^x \cos 4x - \left(\frac{-1}{4}\right) \int e^x \cos 4x\, d$ (I)

$I = \frac{e^x}{4} \sin 4x - \frac{1}{4}\left(\frac{-1}{4} e^x \cos 4x + \frac{1}{4} I\right)$

$I = \frac{e^x}{16}(4 \sin 4x + \cos 4x) - \frac{1}{16} I$

$\frac{17}{16} I = (\qquad\qquad)$

$I = \frac{16}{17}(\qquad\qquad)$

$I = \frac{e^x}{17}(4 \sin 4x + \cos 4x) + C$

Evaluate the trigonometric integral.

3. $\int_0^{\pi/6} \sin^2 x \cos^3 x \, dx$

$= \int_0^{\pi/6} \sin^2 x \cos^2 x \cdot \cos x \, dx$

$= \int_0^{\pi/6} \sin^2 x (1 - \sin^2 x) \cos x \, dx$

$u = \sin x$
$du = \cos x \, dx$

- $x = 0$
  $\Rightarrow u = \sin 0 = 0$
- $x = \pi/6$
  $\Rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$

$\int_0^{1/2} u^2 (1 - u^2) \, du$

$= \int_0^{1/2} (u^2 - u^4) \, du$

$= \frac{u^3}{3} - \frac{u^5}{5} \Big|_0^{1/2}$

$= \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{5}\left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^3 \left(\frac{1}{3} - \frac{1}{5}\left(\frac{1}{2}\right)^2 \right) = \frac{1}{8}\left(\frac{1}{3} - \frac{1}{20}\right) = \frac{1}{8}\left(\frac{20-3}{60}\right)$

$= \frac{17}{480}$

4. $\int \sec^4 x \tan^2 x \, dx$

$= \int \sec^2 x \tan^2 x \cdot \sec^2 x \, dx$

~~$= \int \sec^2 x (\sec^2 x - 1) \sec^2 x \, dx$~~

~~$u = \tan$~~

~~$= \int u^2 (u^2 - 1) \, du$~~

$u = \tan x$
$du = \sec^2 x \, dx$

$= \int (1 + \tan^2 x) \tan^2 x \sec^2 x \, dx$

$= \int (1 + u^2)(u^2) \, du$

$= \int u^2 + u^4 \, du = \frac{u^3}{3} + \frac{u^5}{5} + C$

$= \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + C$

5. Evaluate the integral using a trigonometric substitution.

$$\int \frac{x^2}{\sqrt{x^2-25}}\,dx$$

$x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-25}$

$= \sqrt{(5\sec\theta)^2-25}$

$= \sqrt{25(\sec^2\theta-1)}$

$= \sqrt{25\tan^2\theta} = 5\tan\theta$

$$\int \frac{(5\sec\theta)^2}{5\tan\theta}\cdot 5\sec\theta\tan\theta\,d\theta$$

$= 25\int \sec^3\theta\,d\theta$ Use Reduction Formula

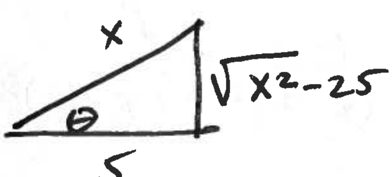
$$\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2}u\tan u + \frac{n-2}{n-1}\int \sec^{n-2}u\,du$$

$25\int \sec^3\theta\,d\theta = 25\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\int \sec\theta\,d\theta\right)$

$n=3$

$= 25\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right)$

$\sec\theta = \frac{x}{5} = \frac{hyp}{adj}$



$\tan\theta = \frac{\sqrt{x^2-25}}{5}$

$$\frac{25}{2}\cdot\frac{x}{5}\cdot\frac{\sqrt{x^2-25}}{5} + \frac{25}{2}\ln\left|\frac{x}{5}+\frac{\sqrt{x^2-25}}{5}\right| + C$$

$$= \frac{x\sqrt{x^2-25}}{2} + \frac{25}{2}\ln\left|\frac{x}{5}+\frac{\sqrt{x^2-25}}{5}\right| + C$$

6. Write out the form of the partial fraction decomposition of the function. Do not determine the numerical values of the coefficients.

(a) $\frac{1}{x^5 - 6x^4 + 9x^3}$ $= \frac{1}{x^3(x^2-6x+9)}$

$= \frac{1}{x^3(x-3)^2}$

$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x-3} + \frac{E}{(x-3)^2}$

(b) $\frac{x-7}{(x+5)^4}$ $= \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{(x+5)^3} + \frac{D}{(x+5)^4}$

(c) $\frac{x^2+5x+1}{(x-1)(x^2+4)^2(x-3)^3}$

$= \frac{A}{x-1} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2} + \frac{F}{x-3} + \frac{G}{(x-3)^2} + \frac{H}{(x-3)^3}$

7. Evaluate the integral.

$$\int \frac{2x^3 - 3x^2 - 1}{x^2 - 3x + 2}\, dx$$

Long division:

$$2x + 3$$

$x^2 - 3x + 2 \,\overline{)\, 2x^3 - 3x^2 + 0x - 1}$

$-2x^3 \pm 6x^2 \mp 4x$

$3x^2 - 4x - 1$

$-3x^2 + 9x \mp 6$

$5x - 7$

$$= \int 2x + 3 + \frac{5x-7}{(x-1)(x-2)}\, dx$$

$$= x^2 + 3x + \int \frac{5x-7}{(x-1)(x-2)}\, dx$$

$$\frac{5x-7}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$5x - 7 = A(x-2) + B(x-1)$$

- $x = 2$

  $3 = B$

- $x = 1$

  $-2 = -A$

  $A = 2$

$$\int \frac{2}{x-1} + \frac{3}{x-2}\, dx = 2\ln|x-1| + 3\ln|x-2|$$

$$x^2 + 3x + 2\ln|x-1| + 3\ln|x-2| + C$$

8. Evaluate the integral.

$$\int \frac{3x^2 - 8x + 36}{x^3 - 6x^2 + 18x}\,dx$$

$$= \frac{3x^2-8x+36}{x(x^2-6x+18)} = \frac{A}{x} + \frac{Bx+C}{x^2-6x+18}$$

$$3x^2-8x+36 = A(x^2-6x+18) + (Bx+C)x$$

$x=0$

$36 = 18A$

$\boxed{A=2}$

$$3x^2-8x+36 = 2(x^2-6x+18) + Bx^2+Cx$$

$$3x^2-8x+36 = 2x^2-12x+36+Bx^2+Cx$$

$$x^2+4x = Bx^2+Cx$$

$\boxed{B=1,\ C=4}$

$$\int \overset{\text{(I)}}{\frac{2}{x}} + \overset{\text{(II)}}{\frac{x+4}{x^2-6x+18}}\,dx$$

(I) $\int \frac{2}{x}\,dx = 2\ln|x| + C_1$

(II) $\int \frac{x+4}{x^2-6x+18}\,dx = \int \frac{x+4}{(x^2-6x+9)+9}\,dx = \int \frac{x+4}{(x-3)^2+9}\,dx$

$u = x-3$, $x = u+3$, $du = dx$

$$\int \frac{u+3+4}{u^2+9}\,du = \int \frac{u}{u^2+9}\,du + \int \frac{4}{u^2+9}\,du$$

subst $w = u^2+9$

$$= \frac{1}{2}\ln|u^2+9| + 4\cdot\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) + C_2 = \frac{1}{2}\ln|x^2-6x+18| + \frac{4}{3}\tan^{-1}\left(\frac{x-3}{3}\right)$$

$$= 2\ln|x| + \frac{1}{2}\ln|x^2-6x+18| + \frac{4}{3}\tan^{-1}\left(\frac{x-3}{3}\right) + C$$

9. How large do we have to choose n so that the approximation to the integral $\int_1^2 \frac{1}{x^2}\,dx$ is accurate to within 0.00001?

(a) For the error of the Midpoint Rule, E_M:

Find K

$f(x) = x^{-2}$

$f'(x) = -2x^{-3}$

$f''(x) = 6x^{-4} = \frac{6}{x^4}$ on $[1,2]$

Max at $x=1$

$K = \frac{6}{1^4} = 6$

$|E_M| \leq \frac{K(b-a)^3}{24n^2} < \frac{1}{10^5}$

$\frac{6(2-1)^3}{24n^2} < \frac{1}{10^5}$

$\frac{10^5}{4} < n^2$

$\sqrt{10^5/4} < n$

$158.1 < n$

$n = 159$

(b) For the error of Trapezoid Rule E_T:

$|E_T| \leq \frac{K(b-a)^3}{12n^2} < \frac{1}{10^5}$

$\frac{6(2-1)^3}{12n^2} < \frac{1}{10^5}$

$\frac{10^5}{2} < n^2$

$\sqrt{\frac{10^5}{2}} < n$

$223.6 < n$

$n = 224$

(c) For the error of Simpson's Rule E_S:

Find K

$f''(x) = 6x^{-4}$

$f'''(x) = -24x^{-5}$

$f^{(4)}(x) = 120x^{-6}$

$= \frac{120}{x^6}$ on $[1,2]$

max at $x=1$

$K = 120$

$|E_S| \leq \frac{K(b-a)^5}{180n^4} \leq \frac{1}{10^5}$

$\frac{120(2-1)^5}{180n^4} \leq \frac{1}{10^5}$

$\frac{2 \cdot 10^5}{3} \leq n^4$

$\sqrt[4]{\frac{2 \cdot 10^5}{3}} \leq n$

$16.06 \leq n$

$n = 18$