$\boxed{\mathcal{A}}$

Math 185 Quiz §7.8 SOLUTIONS

Thursday, 10 March 2010

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\displaystyle\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}}\, dw$

- First solve the improper integral, $\displaystyle\int (2-w)^{-1/2}\, dw$.

$$\begin{aligned} u &= 2-w \\ du &= -dw \\ -du &= dw \\ \int (2-w)^{-1/2}\, dw &= -\int u^{-1/2}\, du = -\frac{u^{1/2}}{1/2} = -2u^{1/2} \\ &= -2(2-w)^{1/2} = -2\sqrt{2-w} + C \end{aligned}$$

- $$\begin{aligned} \int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}}\, dw &= \lim_{t\to-\infty} \int_t^{-1} (2-w)^{-1/2}\, dw \\ &= \lim_{t\to-\infty} \left[-2\sqrt{2-w}\right]_t^{-1} \\ &= \lim_{t\to-\infty} \left[\left(-2\sqrt{2-(-1)}\right) - \left(-2\sqrt{2-t}\right)\right] \\ &= -2\sqrt{3} + \lim_{t\to-\infty} 2\sqrt{2-t} = \infty. \quad \text{The integral is divergent.} \end{aligned}$$

2. $\displaystyle\int_6^8 \frac{4}{(x-6)^3}\, dx$

- First solve the improper integral, $\displaystyle\int \frac{4}{(x-6)^3}\, dx$.

$$\begin{aligned} u &= x-6 \\ du &= dx \\ \int \frac{4}{(x-6)^3}\, dx &= \int 4u^{-3}\, du \\ &= \frac{4u^{-2}}{-2} = -\frac{2}{(x-6)^2} + C \end{aligned}$$

- $$\begin{aligned} \int_6^8 \frac{4}{(x-6)^3}\, dx &= \lim_{t\to6^+} \int_t^8 \frac{4}{(x-6)^3}\, dx \\ &= \lim_{t\to6^+} \left[-\frac{2}{(x-6)^2}\right]_t^8 \\ &= \lim_{t\to6^+} \left[\left(-\frac{2}{(8-6)^2}\right) - \left(-\frac{2}{(t-6)^2}\right)\right] \\ &= -\frac{1}{2} + \lim_{t\to6^+} \frac{2}{(t-6)^2} = \infty. \quad \text{The integral is divergent.} \end{aligned}$$

$\boxed{\mathcal{B}}$

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Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

1. $\displaystyle\int_1^\infty \frac{x+1}{\sqrt{x^2+2x}}\,dx$

- First solve the improper integral, $\displaystyle\int \frac{x+1}{\sqrt{x^2+2x}}\,dx$.

$$\begin{aligned}
u &= x^2+2x \\
du &= (2x+2)\,dx \\
du &= 2(x+1)\,dx \\
\frac{1}{2}du &= (x+1)\,dx \\
\int \frac{x+1}{\sqrt{x^2+2x}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \frac{1}{2}\int u^{-1/2}\,du \\
&= \frac{1}{2}\left(\frac{u^{1/2}}{1/2}\right) + C = (x^2+2x)^{1/2} + C = \sqrt{x^2+2x} + C
\end{aligned}$$

- $$\begin{aligned}
\int_1^\infty \frac{x+1}{\sqrt{x^2+2x}}\,dx &= \lim_{t\to\infty}\int_1^t \frac{x+1}{\sqrt{x^2+2x}}\,dx = \lim_{t\to\infty}\left[\sqrt{x^2+2x}\right]_1^t \\
&= \lim_{t\to\infty}\left[\sqrt{t^2+2t}\right] - \left[\sqrt{(1)1^2+2(1)}\right] = \infty
\end{aligned}$$

The integral is divergent.

2. $\displaystyle\int_2^3 \frac{1}{\sqrt{3-x}}\,dx$

- First solve the improper integral, $\displaystyle\int \frac{1}{\sqrt{3-x}}\,dx$.

$$\begin{aligned}
u &= 3-x \\
du &= -dx \\
-du &= dx \\
\int \frac{1}{\sqrt{3-x}}\,dx &= -\int \frac{1}{\sqrt{u}}\,du = -\int u^{-12}\,du \\
&= -\frac{u^{1/2}}{1/2} + C = -2\sqrt{3-x} + C
\end{aligned}$$

- $$\begin{aligned}
\int_2^3 \frac{1}{\sqrt{3-x}}\,dx &= \lim_{t\to 3^-}\int_2^t \frac{1}{\sqrt{3-x}}\,dx \\
&= \lim_{t\to 3^-}\left[-2\sqrt{3-x}\right]_2^t = \lim_{t\to 3^-}\left[\left(-2\sqrt{3-t}\right) - \left(-2\sqrt{3-2}\right)\right] \\
&= -2\sqrt{3-3} + 2\sqrt{1} = 2
\end{aligned}$$

The integral is convergent. Its value is 2.