Mon. 2/01

# §7.1 Integration by Parts.

HW §7.1 # 1-37 odd

Integration by Parts is a method of Integration.

FORMULA

$$\int u\,dv = uv - \int v\,du$$

Example 1: $\int x \cdot \sin x\,dx$

deriv

$u = x$ | $dv = \sin x\,dx$ | integr
$du = 1 \cdot dx$ | $v = -\cos x$

Put into formula

$= uv - \int v\,du$

$= x \cdot (-\cos x) - \int (-\cos x)\,dx$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + C$

Example $\int x \cdot e^{3x}\,dx$

$u = x$ $\qquad dv = e^{3x}\,dx$

$du = dx$ $\qquad v = \frac{1}{3}e^{3x}$

$= uv - \int v\,du$

$= x\left(\frac{1}{3}e^{3x}\right) - \int \frac{1}{3}e^{3x}\,dx$

$= \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\,dx$

$= \frac{1}{3}xe^{3x} - \frac{1}{3} \cdot \frac{1}{3}e^{3x} + C$

$= e^{3x}\left(\frac{1}{3}x - \frac{1}{9}\right) + C$

Aside

$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$

Let's Derive the formula.

Suppose $u$ and $v$ are differential. Then

$(uv)' = u'v + uv'$ ← product rule

$\int (uv)' = \int (u'v + uv')$

integral and derivative cancel.

$uv = \int u'v + \int uv'$

$uv - \int u'v = \int uv'$

$uv - \int v\,du = \int u\,dv$

$$\int u\,dv = uv - \int v\,du$$

Monday, 2/01

**Example** $\int x \cos 3x \, dx$

$u = x$ $\qquad$ $dv = \cos 3x \, dx$

$du = dx$ $\qquad$ $v = \frac{1}{3} \sin 3x$

$= uv - \int v \, du$

$= x\left(\frac{1}{3} \sin 3x\right)$

$\quad - \int \frac{1}{3} \sin 3x \, dx$

$= \frac{x}{3} \sin 3x - \frac{1}{3}\left(-\frac{1}{3} \sin 3x\right) + C$

$= \boxed{\frac{x}{3} \sin 3x + \frac{1}{9} \overset{\cos(3x)}{\cancel{\sin 3x}} + C}$

FORMULA

- $\int \cos ax \, dx = \frac{1}{a} \sin ax + C$
- $\int \sin ax \, dx = -\frac{1}{a} \cos ax + C$

## Example 2 $\int \ln x \, dx$ Monday, 2/01

If you see $\ln x$, let $u = \ln x$.

$u = \ln x$ $\qquad dv = dx$

$du = \frac{1}{x}$ $\qquad v = x$

$= uv - \int v \, du$

$= (\ln x)(x) - \int x \cdot \frac{1}{x} dx$

$= x \ln x - \int dx$

$= x \ln(x) - x + C$

Example 3 $\int t^2 e^t \, dt$

$u = t^2$ $dv = e^t \, dt$

$du = 2t \, dt$ $v = e^t$

$= uv - \int v \, du$

$= t^2 e^t - \int e^t \cdot 2t \, dt$

$= t^2 e^t - 2 \boxed{\int t e^t \, dt}$

Repeat. Solve using Int. by Parts

$\int t e^t \, dt$

$u = t$ $dv = e^t \, dt$

$du = dt$ $v = e^t$

$= uv - \int v \, du$

$= te^t - \int e^t \, dt$

$= \boxed{te^t - e^t + C}$

Answer:

$t^2 e^t - 2(te^t - et) + C$

$\boxed{e^t (t^2 - 2t + 2) + C}$

# Tabular Integration

We use this for polynomials times $\sin x$, $\cos x$, or $e^x$.

**Example** $\int x^3 \sin x \, dx$

| deriv ↓ | | integrate ↓ |
|---|---|---|
| $x^3$ | $+$ | $\sin x$ |
| $3x^2$ | $-$ | $-\cos x$ |
| $6x$ | $+$ | $-\sin x$ |
| $6$ | $-$ | $\cos x$ |
| $0$ | $+$ | $\sin x$ |
| | | $-\cos x$ |

$$= +x^3(-\cos x) - (3x^2)(-\sin x) + (6x)(\cos x) - (6)(\sin x) + C$$

$$= \boxed{-x^3\cos x + 3x^2 \sin x + 6x\cos x - 6\sin x + C}$$

Example 4 $\int e^x \sin x \, dx$

(exponential times sine or cosine)

$u = e^x$ $\qquad$ $dv = \sin x \, dx$

$du = e^x dx$ $\qquad$ $v = -\cos x$

$= uv - \int v \, du$

$= e^x(-\cos x) - \int (-\cos x) e^x dx$

$= -e^x \cos x + \boxed{\int e^x \cos x \, dx}$

integrate by parts again.

$u = e^x$ $\qquad$ $dv = \cos x \, dx$

$du = e^x dx$ $\qquad$ $v = \sin x$

$= uv - \int v \, du$

$= e^x \sin x - \int \sin x \cdot e^x dx$

$\underbrace{\int e^x \sin x \, dx}_{I} = -e^x \cos x + e^x \sin x - \underbrace{\int e^x \sin x \, dx}_{I}$

$I = -e^x \cos x + e^x \sin x - I$

add $I$ to both sides

$2I = -e^x \cos x + e^x \sin x + C$

$\boxed{I = \frac{1}{2} e^x(-\cos x + \sin x) + C}$

Examples $\int_0^1 \tan^{-1} x \, dx$

$u = \tan^{-1} x$ $\qquad dv = dx$

$du = \frac{1}{1+x^2} dx$ $\qquad v = x$

$= uv - \int v \, du$

$= (\tan^{-1} x)(x) - \boxed{\int x \cdot \frac{1}{1+x^2} dx}$

$\int x \cdot \frac{1}{1+x^2} dx$

use substitution method

$u = 1 + x^2$

$du = 2x \, dx$

$\frac{1}{2} du = x \, dx$

$= \int \frac{1}{u} \cdot \frac{1}{2} du = \frac{1}{2} \ln|u| + C$

$= \frac{1}{2} \ln|1+x^2| + C$

$= \left[ x \tan^{-1} x - \left( \frac{1}{2} \ln|1+x^2| \right) \right]_0^1$

$= \left( (1) \underbrace{\tan^{-1}(1)}_{=\pi/4} - \frac{1}{2} \underbrace{\ln(1+1^2)}_{=\ln 2} \right)$

$- \left( 0 \underbrace{\tan^{-1} 0}_{=0} - \frac{1}{2} \ln(1+0^2) \right)$ $\quad \ln 1 = 0$

$= \boxed{\frac{\pi}{4} - \frac{1}{2} \ln 2}$

§7.1 #9 Tues 2/02

$\int \ln(2x+1)\,dx$

First do substitution.

$w = 2x+1$

$dw = 2\,dx$

$\frac{1}{2}\,dw = dx$

$\frac{1}{2}\int \ln w\,dw$

$u = \ln w \qquad dv = dw$

$du = \frac{1}{w}\,dw \qquad v = w$

$= \frac{1}{2}\left(uv - \int v\,du\right) = \frac{1}{2}\left(w\ln w - \int \frac{1}{w}\cdot \cancel{w}\,dw\right)$

$= \frac{1}{2}\left(w\ln w - \int dw\right)$

$= \frac{1}{2}\left(w\ln w - w + C\right)$

$= \frac{1}{2}(2x+1)\ln\left(2x+1\right) - \frac{1}{\cancel{2}}(\cancel{2}x+1) + C$

$= \frac{1}{2}(2x+1)\ln(2x+1) - x + C$

$-x - \frac{1}{2} + C$

$x \ln$

§7.1 #29

$$\int \cos x \ln(\sin x)\,dx$$

$$= \int \ln(\underbrace{\sin x}_{w}) \cdot \underbrace{\cos x\,dx}_{dw}$$

$w = \sin x$

$dw = \cos x\,dx$

$$= \int \ln w\,dw$$

$u = \ln w \qquad dv = dw$

$du = \frac{1}{w}\,dw \qquad v = w$

$$= uv - \int v\,du = w\ln w - \int \frac{1}{w} w\,du$$

$$= w\ln w - \int dw$$

$$= w\ln w - w$$

$$= \sin x \ln(\sin x) - \sin x + C$$

Tuesday, 2/02

§7.1 #33 $\int \cos\sqrt{x}\, dx$

$w = x^{1/2}$

$dw = \frac{1}{2} x^{-1/2} dx$

$= \int (\cos\sqrt{x}) \, dx$

$dw = \frac{1}{2\sqrt{x}} dx$, where $\sqrt{x} = w$

$= 2\int w \cos w \, dw$

$2\,dw = \frac{1}{\sqrt{x}} dx$, where $\sqrt{x} = w$

$2w\,dw = dx$

| | | |
|---|---|---|
| $w$ | $+$ | $\cos w$ |
| $1$ | $-$ | $\sin w$ |
| $0$ | $+$ | $-\cos w$ |

$= w \sin w - (1)(-\cos w) + C$

$= \sqrt{x} \sin\sqrt{x} + \cos\sqrt{x} + C$

Example 6 Prove the reduction formula.

$$\int \sin^n x\,dx = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$n \geq 2$

Aside: Use the reduction formula to evaluate

$$\int \sin^4 x\,dx = \frac{-1}{4}\cos x \sin^{3} x + \frac{3}{4}\int \sin^{2} x\,dx$$

$n-1=3$ $n=3$ $n-2=2$

$n=4$ $n=4$ $n=4$

$$= \frac{-1}{4}\cos x \sin^3 x + \frac{3}{4}\left(\int \sin^2 x\,dx\right)$$

$n=2$

$$= -\frac{1}{4}\cos x \sin^3 x + \frac{3}{4}\left(-\frac{1}{2}\cos x \sin x + \frac{1}{2}\int dx\right)$$

$$= -\frac{1}{4}\cos x \sin^3 x - \frac{3}{8}\cos x \sin x + \frac{3}{8}x + C$$

Let's prove the reduction formula.

$$I = \int \sin^n x\, dx = \int \sin^{n-1} x \, \sin x \, dx$$

Int. by Parts.

$$u = (\sin x)^{n-1} \qquad dv = \sin x\, dx$$

$$du = (n-1)(\sin x)^{n-2} \cos x\, dx \qquad v = -\cos x$$

$$= uv - \int v\, du$$

$$= -\sin^{n-1} x \cos x - \int (n-1) \sin x^{n-2} \cos x (-\cos x)\, dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \underbrace{\cos^2 x}_{(1-\sin^2 x)}\, dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x (1 - \sin^2 x)\, dx$$

$$I = -\sin^{n-1} x \cos x + (n-1) \left( \int \sin^{n-2} x\, dx - \underbrace{\int \sin^n x\, dx}_{I} \right)$$

$$I = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x\, dx - (n-1) I$$

$$\underbrace{I + (n-1) I}_{nI} = -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x\, dx$$

$$I = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x\, dx$$

Weds 3/03

#25 $\int_0^1 \frac{y}{e^{2y}}\,dy$

$= \int_0^1 y e^{-2y}\,dy$

$u = y$ $\quad dv = e^{-2y}\,dy$

$du = dy$ $\quad v = -\frac{1}{2}e^{-2y}$

$= uv - \int v\,du$

$= -\frac{1}{2}ye^{-2y} + \int +\frac{1}{2}e^{-2y}\,dy$

$= -\frac{1}{2}ye^{-2y} + \frac{1}{2}\left(-\frac{1}{2}e^{-2y}\right) + C$

$= \left[-\frac{1}{2}ye^{-2y} - \frac{1}{4}e^{-2y}\right]_0^1$

Aside $\int e^{-2y}\,dy$

$u = -2y$

$du = -2\,dy$

$-\frac{1}{2}du = dy$

$= -\frac{1}{2}\int e^u\,du$

$= -\frac{1}{2}e^u + C$

$= -\frac{1}{2}e^{-2y} + C$

$$= \left( -\frac{1}{2}(1)e^{-2(1)} - \frac{1}{4}e^{-2(1)} \right) - \left( -\frac{1}{2}\cdot 0 \cdot e^{-2\cdot 0} - \frac{1}{4}e^{-2\cdot 0} \right)$$

$$= -\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2} + \frac{1}{4}$$

$$= -\frac{3}{4}e^{-2} + \frac{1}{4}$$

§7.1 

#13 $\int t \sec^2 2t \, dt$

$u = t$ $\qquad$ $dv = \sec^2(2t)\,dt$

$du = dt$ $\qquad$ $v = \frac{1}{2}\tan(2t)$

$= uv - \int v\,du$

$= \frac{1}{2} t \tan(2t) - \int \frac{1}{2}\tan(2t)\,dt$

$$\boxed{= \frac{1}{2} t \cdot \tan(2t) - \frac{1}{2}\cdot\frac{1}{2}\ln|\sec(2t)| + C}$$

FORMULA

$\int \tan u\,du = \ln|\sec u| + C$

Pf

$\int \tan \cancel{x}\, du = \int \frac{\sin \cancel{x}}{\cos \cancel{x}}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$-\int \frac{1}{u}\,du = -\ln|u| + C$

$= -1 \ln|u| + C$

$= \ln|u|^{-1} + C$

$= \ln\frac{1}{|u|} + C$

$= \ln\frac{1}{|\cos x|} + C$

$= \ln|\sec x| + C$

Use (a)ln x = ln $x^a$

# §7.2 Trigonometric Integrals

HW §7.2 # 1-49 odd

Example:

$$\int \underbrace{\sin^5 x}_{(\sin x)^5} \overbrace{\cos x \, dx}^{du}$$

We use u-substitution.

$u = \sin x$

$du = \cos x \, dx$

$$\int u^5 \, du = \frac{u^6}{6} + C$$

$$= \frac{\sin^6 x}{6} + C$$

Example

$$\int \sin^4 x \cos^3 x \, dx$$

$$\boxed{\begin{aligned} u &= \sin x \\ du &= \cos x \, dx \end{aligned}}$$

$$= \int \underbrace{\sin^4 x}_{u^4} \; \underbrace{\cos^2 x}_{\underbrace{(1-\sin^2 x)}_{(1-u^2)}} \cdot \underbrace{\cos x \, dx}_{du}$$

$$= \int u^4 (1-u^2) \, du$$

$$= \int (u^4 - u^6) \, du$$

$$= \frac{u^5}{5} - \frac{u^7}{7} + C$$

$$= \boxed{\frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C}$$

Aside

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

## Example

$$\int \sin^4 x \cos^5 x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \underbrace{\sin^4 x}_{u^4} \cos^4 x \cdot \underbrace{\cos x \, dx}_{du}$$

$\cos^4 x$

$(\cos^2 x)^2$

$(1 - \sin^2 x)^2$

$(1 - u^2)^2$

$$= \int u^4 (1-u^2)^2 \, du$$

$$= \int u^4 (1 - 2u^2 + u^4) \, du$$

$$= \int u^4 - 2u^6 + u^8 ) \, du$$

$$= \frac{u^5}{5} - 2\frac{u^7}{7} + \frac{u^9}{9} + C$$

$$= \boxed{\frac{\sin^5 x}{5} - \frac{2}{7} \sin^7 x + \frac{\sin^9 x}{9} + C}$$

Wednesday, 2/03

# Example $\int \sin^2 x \, dx$

**Trig Identities**

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

Proof:

$$\begin{aligned}
\cos 2x &= \cos^2 x - \sin^2 x \\
\cos 2x &= \cos^2 x - (1-\cos^2 x) \\
\cos 2x &= 2\cos^2 x - 1 \\
1+\cos 2x &= 2\cos^2 x \\
\frac{1}{2}(1+\cos 2x) &= \cos^2 x \\
\cos^2 x &= \frac{1}{2}(1+\cos 2x)
\end{aligned}$$

$$= \int \frac{1-\cos 2x}{2} \, dx$$

$$= \frac{1}{2}\int (1-\cos 2x) \, dx$$

$$= \frac{1}{2}\left(x + -\frac{1}{2}\sin(2x)\right) + C$$

FORMULA

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax + $$

**Example** $\int \sin^4 x \, dx$

$$= \int (\sin^2 x)^2 \, dx$$

$$= \int \left( \frac{1 - \cos 2x}{2} \right)^2 dx$$

$$= \int \frac{1 - 2\cos 2x + \cos^2(2x)}{4} \, dx$$

$$= \frac{1}{4} \int \left( 1 - 2\cos 2x + \frac{1 + \cos 2(2x)}{2} \right) dx$$

$$= \frac{1}{4} \int \left( 1 - 2\cos 2x + \frac{1}{2} + \frac{1}{2}\cos(4x) \right) dx$$

$\frac{3}{2}x$

$$= \frac{1}{4} \left( x - 2\frac{1}{2}\sin 2x + \frac{1}{2}x + \frac{1}{2} \cdot \frac{1}{4} \sin 4x \right) + C$$

$$= \frac{1}{4} \left( \frac{3}{2}x - \sin 2x + \frac{1}{8} \sin 4x \right) + C$$

How about using the reduction formula?

Example $\int \sin^2 x \cos^2 x \, dx$

both powers of $\sin x$ & $\cos x$ are even.

$= \int \sin^2 x (1 - \sin^2 x) \, dx$

Now we have all even powers of $\sin x$.

$= \int (\sin^2 x - \sin^4 x) \, dx$

We then plug in the answers from the last two examples.

Example $\int \sin^6 x\,dx = \int (\sin^2 x)^3\,dx$

Use the reduction formula

$$\int \sin^n x\,dx = \frac{-1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$n=6$

$$\int \sin^6 x\,dx = \frac{-1}{6}\cos x \sin^5 x + \frac{5}{6}\int \underbrace{\sin^4 x\,dx}_{n=4}$$

$$= \frac{-1}{6}\cos x \sin^5 x + \frac{5}{6}\left[\frac{-1}{4}\cos x \sin^3 x + \frac{3}{4}\int \underbrace{\sin^2 x\,dx}_{n=2}\right]$$

$$= -\frac{1}{6}\cos x \sin^5 x - \frac{5}{24}\cos x \sin^3 x + \frac{5}{8}\left(\frac{-1}{2}\cos x \sin x + \frac{1}{2}\int dx\right)$$

$$= \frac{-1}{6}\cos x \sin^5 x - \frac{5}{24}\cos x \sin^3 x - \frac{5}{16}\cos x \sin x + \frac{5}{16}x + C$$

§7.2 Thurs 2/04

# Powers of Secant and Tangent.

Example: $\int \underbrace{\tan^4 x}_{u^4} \underbrace{\sec^2 x\,dx}_{du}$

$u = \tan x$
$du = \sec^2 x\,dx$

$$\int u^4\,du = \frac{u^5}{5} + C$$
$$= \frac{\tan^5 x}{5} + C$$

Example: $\int \tan^4 x \sec^4 x\,dx$

$u = \tan x$
$du = \sec^2 x\,dx$

$$= \int \underbrace{\tan^4 x}_{u^4} \sec^2 x \cdot \underbrace{\sec^2 x\,dx}_{du}$$

$$\boxed{\begin{aligned} &= 1 + \tan^2 x \\ &= 1 + u^2 \end{aligned}}$$

If $\sec x$ is raised to an even power, we can let $u = \tan x$

$$= \int u^4 (1 + u^2)\,du$$
$$= \int (u^4 + u^6)\,du$$
$$= \frac{u^5}{5} + \frac{u^7}{7} + C$$
$$= \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C$$

$$\boxed{\underline{\text{Identity}}: \quad 1+\tan^2 x = \sec^2 x}$$

<u>Example</u>: $\int \tan^3 x \sec^5 x \, dx$

$$\boxed{\begin{aligned} u &= \sec x \\ du &= \sec x \tan x \, dx \end{aligned}}$$

$$= \int \tan^2 x \underbrace{\sec^4 x}_{u^4} \cdot \underbrace{\sec x \tan x \, dx}_{du}$$

$$\tan^2 x = (\sec^2 x - 1)$$
$$= (u^2 - 1)$$

$$= \int (u^2-1) u^4 \, du$$
$$= \int (u^6 - u^4) \, du$$
$$= \frac{u^7}{7} - \frac{u^5}{5} + C$$
$$= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

If $\tan x$ is raised to an odd power, let $u = \sec x$.

<u>Recall</u>:

$$\int \tan x \, dx = \ln|\sec x| + C$$

What if secant is <u>not</u> to an even power <u>and</u> tangent is <u>not</u> to an odd power?

<u>Example</u> $\int \tan^2 x \sec x \, dx$

<u>Step 1</u>: Write everything in terms of $\sec x$.
Use $\tan^2 x = \sec^2 x - 1$

$$\int (\sec^2 x - 1) \sec x \, dx$$
$$= \int (\sec^3 x - \sec x) \, dx$$
$$= \text{I} \int \sec^3 x \, dx - \text{II} \int \sec x \, dx$$

II FORMULA $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

<u>Pf</u> $\int \sec x \, dx$

$$= \int \sec x \left( \frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$$

$u = \sec x + \tan x$
$du = (\sec x \tan x + \sec^2 x) \, dx$

$$= \int \frac{1}{u} \, du = \ln|u| + C$$
$$= \ln|\sec x + \tan x| + C.$$

# (I) Reduction FORMULA

§7.1 #50

$$\int \sec^n x\,dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2} x\,dx$$

$n \neq 1$

$$\int \sec^3 x\,dx = \frac{\tan x \sec^{3-2} x}{3-1} + \frac{3-2}{3-1}\int \sec^{3-2} x\,dx$$

$n=3$

$$= \frac{\tan x \sec x}{2} + \frac{1}{2}\int \sec x\,dx$$

$$= \frac{\tan x \sec x}{2} + \frac{1}{2}\ln|\sec x + \tan x| + C$$

Final Answer: $\quad \mathrm{I} - \mathrm{II} =$

$$= \overset{\mathrm{I}}{\left(\frac{\tan x \sec x}{2} + \frac{1}{2}\ln|\sec x + \tan x|\right)} - \overset{\mathrm{II}}{\left(\ln|\sec x + \tan x|\right)} + C$$

$$= \frac{\tan x \sec x}{2} - \frac{1}{2}\ln|\sec x + \tan x| + C$$

# Strategy for Evaluating $\int \tan^m x \sec^n x\,dx$

1) If the power of secant is even,
   let $u = \tan x$

2) If the power of $\tan x$ is odd,
   let $u = \sec x$

3) If the power of $\sec x$ is odd
   and the power of $\tan x$ is even,
   - Write everything in terms of $\sec x$.
     Use $\tan^2 x = \sec^2 x - 1$
   - Use the reduction formula for $\int \sec^n x\,dx$
     and $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

**Example** Evaluate

$$\int \sin 4x \cos 5x \, dx$$

FORMULA

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$A = 4x, \quad B = 5x$

$$= \frac{1}{2}\int\left(\sin(4x-5x) + \sin(4x+5x)\right)dx$$

$$= \frac{1}{2}\int\left[\sin(-x) + \sin(9x)\right]dx$$

$$= \frac{1}{2}\int\left(-\sin x + \sin 9x\right)dx$$

Even & Odd Identies

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$$= \frac{1}{2}\left[-(-\cos x) + \frac{-1}{9}\cos(9x)\right] + C$$

$$= \frac{1}{2}\left[\cos x - \frac{1}{9}\cos(9x)\right] + C$$

$$\boxed{\text{Quiz Thurs. } 2/11 \quad \S 7.1, 7.2}$$