Monday 2/08 **Math 185**

§7.2 #25 $\int \sec^6 t\, dt$

even power of ~~secant~~. Let $u = \tan t$.

Math 185 Notes
Monday, 2/08/10
Koines

$$= \int \sec^4 \cancel{t} \cdot \sec^2 t$$

$$= \int (\sec^2 t)^2 \cdot \sec^2 t$$

$u = \tan t$

$du = \sec^2 t\, dt$

$$= \int (1 + \tan^2 t)^2 \cdot \sec^2 t$$

$1 + \tan^2 t = \sec^2 t$

$$= \int (1 + u^2)^2\, du$$

$$= \int (1 + 2u^2 + u^4)\, du$$

$$= u + 2\frac{u^3}{3} + \frac{u^5}{5} + C$$

$$= \tan t + \frac{2}{3}\tan^3 t + \frac{1}{5}\tan^5 t + C$$

§7.2 #31 $\int \tan^5 x \, dx$

$= \int \tan^4 x \cdot \tan x \, dx$

- odd power of tangent.
- even power of secant

$= \int (\tan^2 x)^2 \tan x \, dx$

$= \int (\sec^2 x - 1)^2 \tan x \, dx$ $u=$

$= \int (\sec^4 x - 2\sec^2 x + 1) \tan x \, dx$

$= \int (\sec^4 x - 2\sec^2 x) \tan x \, dx + \int \tan x \, dx$

I $\int (\sec^4 x - 2\sec^2 x) \tan x \, dx$

$= \int (\sec^3 x - 2\sec x) \cdot \sec x \tan x \, dx$

$u = \sec x$

$du = \sec x \tan x \, dx$

$= \int (u^3 - 2u) \, du = \frac{u^4}{4} - \frac{2u^2}{2} + C_1$

$= \frac{\sec^4 x}{4} - \sec^2 x + C_1$

II $\int \tan x \, dx$

$= \ln|\sec x| + C_2$

Answer: $\frac{\sec^4 x}{4} - \sec^2 x + \ln|\sec x| + C$

$-(1 + \tan^2 x)$

or

$= \frac{\sec^4 x}{4} - \tan^2 x + \ln|\sec x| + C$

§7.2 #41 $\int \csc x \, dx$

$$= \int \csc x \left(\frac{+\csc x + \cot x}{+\csc x + \cot x} \right) dx$$

$$= \int \frac{+\csc^2 x + \csc x \cot x}{+\csc x + \cot x} \, dx$$

Substitution

$$u = \pm\csc x + \cot x$$
$$du = (\mp \csc x \cot x - \csc^2 x) \, dx$$
$$\mp du = (\csc x \cot x \pm \csc^2 x) \, dx$$

$$= \mp \int \frac{1}{u} \, du = \mp \ln|u| + C$$
$$= \mp \ln|\csc x \pm \cot x| + C$$
$$= (-1) \cdot \ln|\csc x + \cot x| + C$$

$$= \ln|\csc x + \cot x|^{-1} + C$$

$$= \ln\left|\frac{1}{\csc x + \cot x}\right| + C$$

$$= \ln\left|\frac{1}{(\csc x + \cot x)} \cdot \frac{\csc x - \cot x}{\csc x - \cot x}\right| + C$$

$$= \ln\left|\frac{\csc x - \cot x}{\underbrace{\csc^2 x - \cot^2 x}_{=1}}\right| + C$$

$$= \ln|\csc x - \cot x| + C$$

$$\cot^2 x + 1 = \csc^2 x$$
$$\Rightarrow \csc^2 x - \cot^2 x = 1$$

$$\boxed{\text{FORMULA} \quad \int \csc x \, dx = \ln|\csc x - \cot x| + C}$$

§7.2 #49 $\int t \sec^2(t^2)\tan^4(t^2)\,dt$

$\frac{1}{2}du$

$u = t^2$
$du = 2t\,dt$
$\frac{1}{2}du = t\,dt$

$= \frac{1}{2}\int \sec^2(u)\tan^4(u)\,du$

dw

$w = \tan u$
$dw = \sec^2 u\,du$

$= \frac{1}{2}\int w^4\,dw$

$= \frac{1}{2}\frac{w^5}{5} + C = \frac{\tan^5 u}{10} + C$

$= \frac{\tan^5(t^2)}{10} + C$

§7.3 Trigonometric Substitution.

Hw §7.3 #1-29 odd

Example $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\,d\theta$

Aside

$\frac{x}{3} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

Focus on:

$$\sqrt{9-x^2} = \sqrt{9-(3\sin\theta)^2}$$
$$= \sqrt{9-9\sin^2\theta}$$
$$= \sqrt{9(1-\sin^2\theta)}$$
$$= \sqrt{9\cos^2\theta} = 3\cos\theta$$

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = \int \frac{3\cos\theta}{(3\sin\theta)^2}\cdot 3\cos\theta\,d\theta$$
$$= \int \frac{\cancel{9}\cos^2\theta}{\cancel{9}\sin^2\theta}\,d\theta = \int \cot^2\theta\,d\theta$$
$$= \int (\csc^2\theta - 1)\,d\theta$$
$$= -\cot\theta - \theta + C$$

substitute back for x.

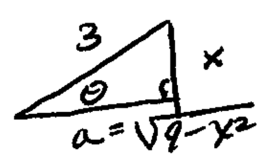
$x = 3\sin\theta$

$\sin\theta = \frac{x}{3}$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

Find $\cot\theta$

$\sin\theta = \frac{x}{3} = \frac{\text{opp}}{\text{hyp}}$



$a^2 + x^2 = 3^2$

$a^2 = 9 - x^2$

$a = \sqrt{9-x^2}$

$$\cot\theta = \frac{adj}{\cancel{hyp}\ opp} = \frac{\sqrt{9-x^2}}{x}$$

Answer: $-\cot\theta - \theta + C$

$$= -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C$$

Example $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

Use:
$\tan^2\theta + 1 = \sec^2\theta$

$x = 2\tan\theta$
$dx = 2\sec^2\theta\, d\theta$

$$\sqrt{x^2+4} = \sqrt{(2\tan\theta)^2+4}$$
$$= \sqrt{4\tan^2\theta+4}$$
$$= \sqrt{4(\tan^2\theta+1)} = \sqrt{4\sec^2\theta}$$
$$= 2\sec\theta$$

$$= \int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$
$$= \int \frac{1}{(2\tan\theta)^2\, 2\sec\theta} \cdot 2\sec^2\theta\, d\theta$$
$$= \int \frac{\cancel{2}\sec^2\theta}{4\tan^2\theta \cdot \cancel{2}\sec\theta}\, d\theta$$
$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

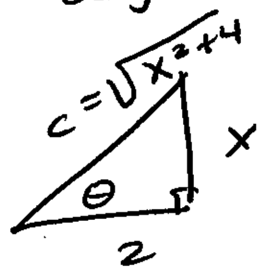
Write in terms of $\cos\theta, \sin\theta$

$$= \int \frac{1}{4}\cdot \sec\theta \cdot \frac{1}{\tan^2\theta}\, d\theta = \frac{1}{4}\int \frac{1}{\cos\theta} \cdot \frac{1}{\left(\frac{\sin^2\theta}{\cos^2\theta}\right)}\, d\theta$$
$$= \frac{1}{4}\int \frac{1}{\cos\theta}\, \frac{\cos^2\theta}{\sin^2\theta}\, d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$
$$= \frac{1}{4}\int \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta}\, d\theta = \frac{1}{4}\int \cot\theta \cdot \csc\theta\, d\theta$$
$$= -\frac{1}{4}\csc\theta + C$$

Write in terms of x.

$x = 2\tan\theta$

$\tan\theta = \frac{x}{2} = \frac{\text{opp}}{\text{adj}}$



$c^2 = x^2 + 2^2$

$c = \sqrt{x^2+4}$

$\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{x}{\sqrt{x^2+4}}$

$\csc\theta = \frac{\sqrt{x^2+4}}{x}$

Answer: $-\frac{1}{4}\,\frac{\sqrt{x^2+4}}{x} + C$

Table

	Substitution	Identity
$\sqrt{a^2-x^2}$	$x = a\sin\theta$	$1-\sin^2\theta = \cos^2\theta$
$\sqrt{a^2+x^2}$	$x = a\tan\theta$	$1+\tan^2\theta = \sec^2\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2$