Tues. 2/16/2010 Math 185

§7.4 #19 $\int \frac{1}{(x+5)^2(x-1)}\,dx$

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{x-1}$$

$$1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2$$

- $x = 1$

  $1 = C(1+5)^2$

  $1 = 36C,\quad \boxed{C = \frac{1}{36}}$

- $x = -5$

  $1 = B(-5-1)$

  $-6B = 1,\quad \boxed{B = \frac{-1}{6}}$

- $x = 0$

  $1 = A(0+5)(0-1) + B(0-1) + C(0+5)^2$

  $1 = -5A - B + 25C$

  $1 = -5A - \left(-\frac{1}{6}\right) + 25\left(\frac{1}{36}\right)$

  Multiply through by LCD

  $36 = -5 \cdot 36A + 6 + 25$

  $36 = -5 \cdot 36A + 31$

  $-36 \cdot \cancel{5}A = \cancel{5} \cdot 1$

  $\boxed{A = \frac{-1}{36}}$

$$= \int \left[\frac{-1}{36} \cdot \left(\frac{1}{x+5} \right) + \frac{-1}{6} (x+5)^{-2} + \frac{1}{36} \left(\frac{1}{x-1} \right) \right] dx$$

$$= \frac{-1}{36} \ln|x+5| + \frac{-1}{6} \frac{(x+5)^{-1}}{-1} + \frac{1}{36} \ln|x-1| + C$$

$$= \frac{-1}{36} \ln|x+5| + \frac{1}{6(x+5)} + \frac{1}{36} \ln|x-1| + C$$

§7.3 #13 $\displaystyle\int \frac{\sqrt{x^2-9}}{x^3}\,dx$

$$\boxed{\begin{aligned} x &= 3\sec\theta \\ dx &= 3\sec\theta\tan\theta\,d\theta \end{aligned}}$$

$$\begin{aligned} \sqrt{x^2-9} &= \sqrt{(3\sec\theta)^2-9} \\ &= \sqrt{9\sec^2\theta-9} \\ &= \sqrt{9(\sec^2\theta-1)} \\ &= \sqrt{9\tan^2\theta} = 3\tan\theta \end{aligned}$$

$$= \int \frac{3\tan\theta \cdot 3\sec\theta\tan\theta\,d\theta}{(3\sec\theta)^3}$$

$$= \int \frac{9\tan^2\theta\sec\theta}{27\sec^3\theta}\,d\theta = \frac{1}{3}\int \frac{\tan^2\theta}{\sec^2\theta}\,d\theta$$

$$= \frac{1}{3}\int \tan^2\theta \cdot \frac{1}{\sec^2\theta}\,d\theta = \frac{1}{3}\int \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \cos^2\theta\,d\theta$$

$$= \frac{1}{3}\int \sin^2\theta\,d\theta = \frac{1}{3}\int \frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$= \frac{1}{6}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

Write everything in terms of x.

$x = 3\sec\theta$, $\quad x = 3 \cdot \frac{1}{\cos\theta}$

$\boxed{\sec\theta = \frac{x}{3}}$, $\quad \frac{x}{3} = \frac{1}{\cos\theta}$, $\quad \boxed{\cos\theta = \frac{3}{x}}$

- Work on θ

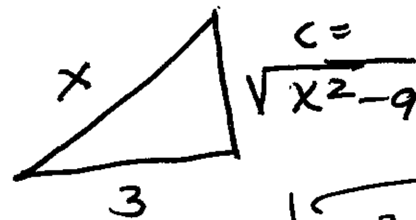
$$\sec\theta = \frac{x}{3}$$

$$\boxed{\theta = \sec^{-1}\left(\frac{x}{3}\right)}$$

- ~~Wor~~ Work on $\sin 2\theta$

$$\sin 2\theta = 2\cos\theta\sin\theta$$

Use $\cos\theta = \frac{3}{x} = \frac{adj}{hyp}$



$$\sin 2\theta = 2\left(\frac{3}{x}\right)\left(\frac{\sqrt{x^2-9}}{\cancel{3}x}\right)$$

$$3^2 + c^2 = x^2$$
$$c^2 = x^2 - 3^2$$
$$c = \sqrt{x^2 - 3^2}$$
$$c = \sqrt{x^2 - 9}$$

$$\sin\theta = \frac{opp}{hyp} = \frac{\sqrt{x^2-9}}{\cancel{3}x}$$

Answer:

$$\frac{1}{6}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

$$= \frac{1}{6}\left[\sec^{-1}\left(\frac{x}{3}\right) - \frac{1}{\cancel{2}}\left(\frac{\cancel{2}\cdot 3\sqrt{x^2-9}}{x^2}\right)\right] + C$$

$$= \boxed{\frac{1}{6}\sec^{-1}\left(\frac{x}{3}\right) - \frac{\sqrt{x^2-9}}{2x^2} + C}$$

§7.4 #39 Rational Substitution.

$$\int \frac{1}{x\sqrt{x+1}}\,dx$$

Let u equal the radical expression.

$u=\sqrt{x+1}$

$u^2=x+1 \rightarrow x=u^2-1$

$2u\,du=dx$

$$=\int \frac{1}{(u^2-1)(\cancel{u})}\cdot 2\cancel{u}\,du$$

$$=\int \frac{2}{u^2-1}\,du$$

Partial Fractions

$$\frac{2}{u^2-1}=\frac{2}{(u-1)(u+1)}=\frac{A}{u-1}+\frac{B}{u+1}$$

$$2=A(u+1)+B(u-1)$$

$$2=Au+A+Bu-B$$

$$0\cdot u+2=(A+B)u+(A-B)$$

$$A+B=0$$
$$A-B=2$$
$$2A=2,\quad \boxed{A=1}$$

$$A+B=0$$
$$B=-A$$
$$\boxed{B=-1}$$

$$=\int\left(\frac{1}{u-1}-\frac{1}{u+1}\right)du$$

$$=\ln|u-1|-\ln|u+1|+C$$

$$=\ln|\sqrt{x+1}-1|-\ln|\sqrt{x+1}+1|+C$$

Use $\ln A-\ln B=\ln\frac{A}{B}$ if you want

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$$\S 7.4\ \#41)\ \int_9^{16} \frac{\sqrt{x}}{x-4}\,dx$$

Rational Subst.

$u = \sqrt{x}$

$u^2 = x$

$2u\,du = dx$

when $x = 9$,

$u = \sqrt{9} = 3$

when $x = 16$

$u = \sqrt{16} = 4$

$$= \int_3^4 \frac{u}{u^2-4} \cdot 2u\,du$$

$$= 2\int_3^4 \frac{u^2}{u^2-4}\,du$$

deg num = deg denom.

$$= 2\int_3^4 \frac{(u^2-4)+4}{(u^2-4)}\,du$$

$$= 2\int_3^4 \left(\frac{u^2-4}{u^2-4} + \frac{4}{u^2-4}\right)du$$

$$= 2\int_3^4 \left(1 + \frac{4}{u^2-4}\right)du$$

$$\begin{array}{r} 1 + \frac{4}{u^2-4} \\ u^2-4\ \overline{)\ u^2+0u+0} \\ \underline{-u^2 \quad\quad +4} \\ 4 \end{array}$$

Aside

$$\int \frac{4}{u^2-4}\,du =$$

$$\frac{4}{u^2-4} = \frac{4}{(u-2)(u+2)} = \frac{A}{u-2} + \frac{B}{u+2}$$

$$4 = A(u+2) + B(u-2)$$

- $u = 2$

  $4 = A(2+2)$

  $4A = 4,\quad \boxed{A = 1}$

- $u = -2$

  $4 = B(-2-2)$

  $-4B = 4\quad \boxed{B = -1}$

$$\int \left(\frac{1}{u-2} - \frac{1}{u+2} \right) du$$

$$= \ln|u-2| - \ln|u+2|$$

Final Answer:

$$2\int_3^4 \left(1 + \frac{4}{u^2-4} \right) du = 2\Big[u + \ln|u-2| - \ln|u+2| \Big]_3^4$$

$$= 2\big[4 + \ln|4-2| - \ln|4+2|\big]$$
$$- 2\big[3 + \ln|3-2| - \ln|3+2|\big]$$

$$= 2\big[4 + \ln 2 - \ln 6\big]$$
$$-2\big[3 + \underbrace{\ln 1}_{0} - \ln 5\big]$$

$$= 2\big[4 + \ln 2 - \ln 6 - 3 + \ln 5\big]$$

$$= \boxed{2\big[1 + \ln 2 - \ln 6 + \ln 5\big]}$$

$$= 2\Big[1 + \ln \frac{\cancel{2}\cdot 5}{\cancel{6}\,3}\Big]$$

$$= 2 + 2\ln\frac{5}{3} = 2 + \ln\left(\frac{5}{3}\right)^2$$

$$= \boxed{2 + \ln\frac{25}{9}}$$

Another way to solve for A & B.

$$4 = A(u+2) + B(u-2)$$

$$\cancel{4 = Au + 8 + Bu - 2}$$

$$4 = Au + 2A + Bu - 2B$$

$$4 = (A+B)u + (2A - 2B)$$

$$A + B = 0$$
$$2A - 2B = 4$$

$$A + B = 0$$
$$A - B = 2$$
$$2A = 2$$
$$\boxed{A = 1}$$

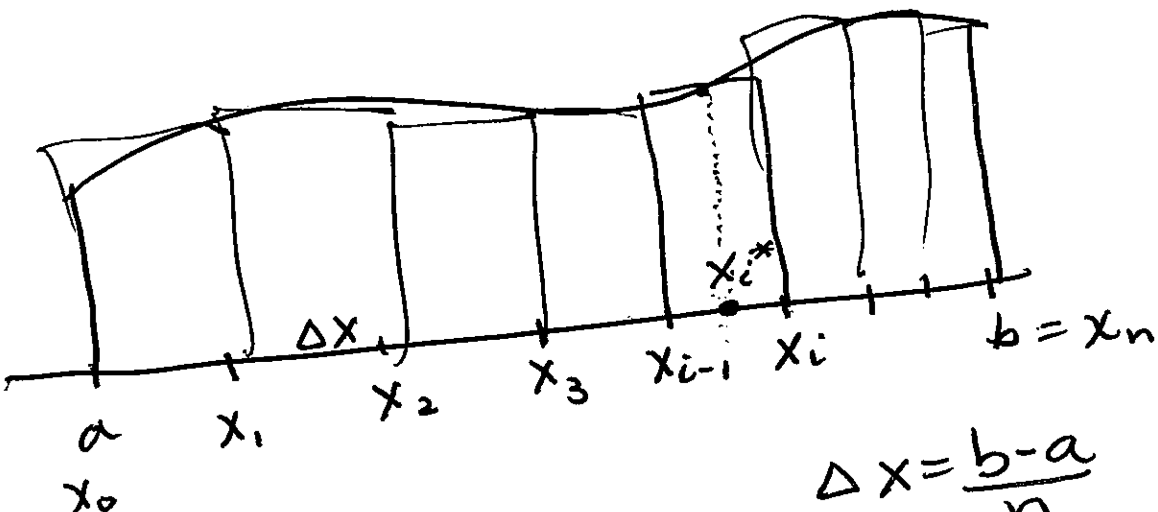
$$A + B = 0$$
$$B = -A$$
$$\boxed{B = -1}$$

§7.7 Approximate Integration

Hw §7.7 # 7 – 21 odd

By definition.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

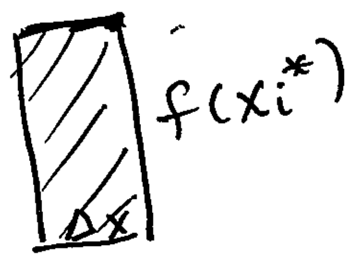


$$\Delta x = \frac{b-a}{n}$$

$x_{i-1} \le x_i^* \le x_i$

x_i^* determines the height of the rectangle, $f(x_i^*)$

Area of one rectangle is $f(x_i^*)\,\Delta x$



The Midpoint Rule to Approximate $\int_a^b f(x)\,dx$

$$\int_a^b f(x)\,dx \approx M_n = \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + \dots + f(\bar{x}_n)\right]$$

$$\Delta x = \frac{b-a}{n}$$

$$\bar{x}_i = \tfrac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$$