Math 185

Next Thurs. 2/25

Test 7.1 – 7.4, 7.7

Homework Review.

§7.4 # 47 $\displaystyle\int \frac{e^{2x}}{e^{2x}+3e^{x}+2}\,dx$

$$= \int \frac{(e^x)^2}{(e^x)^2+3(e^x)+2}\,dx$$

Note $(e^x)^2 = e^{2x}$

$u = e^x$

$du = e^x\,dx$

$$= \int \frac{\overset{=u}{(e^x)}}{\underset{u}{(e^x)^2}+3\underset{u}{(e^x)}+2} \cdot \underbrace{e^x\,dx}_{du}$$

$$= \int \frac{u}{u^2+3u+2}\,du$$

$$\frac{u}{u^2+3u+2} = \frac{u}{(u+1)(u+2)} = \frac{A}{u+1} + \frac{B}{u+2}$$

$u = A(u+2) + B(u+1)$

- $u = -1$

  $-1 = A(-1+2) + B(-1+1)$

  $\boxed{A = -1}$

- $u = -2$

  $-2 = A(-2+2) + B(-2+1)$

  $-2 = -B$, $\boxed{B = 2}$

ERROR Bound FOR Simpson's Rule

Suppose that $|f^{(4)}(x)| \le K$ for $a \le x \le b$. If E_S is the error involved in using Simpson's Rule, then

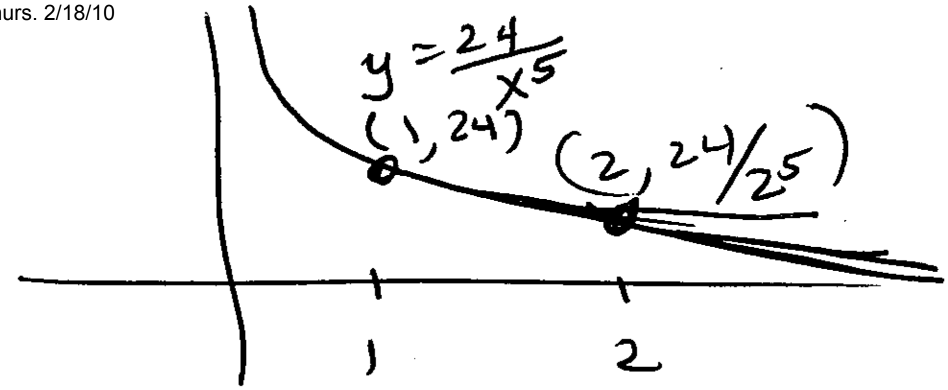
$$|E_S| \le \frac{K(b-a)^5}{180n^4}$$

EXAMPLE 6: How large should we take n in order to guarantee that the Simpson's Rul approximation for $\int_1^2 \left(\frac{1}{x}\right)dx$ is accurate to within 0.0001?

SOLUTION: Find K.

$$f(x) = x^{-1}$$
$$f'(x) = -1 \cdot x^{-2}$$
$$f''(x) = 2x^{-3}$$
$$f'''(x) = -6x^{-4}$$
$$f^{(IV)}(x) = 24x^{-5}, \quad 1 \le x \le 2$$

Find max value of $y = \frac{24}{x^5}$ on $[1, 2]$.



Max value of $|f^{(4)}(x)|$ on $[1, 2]$ is 24.

Let $K = 24$.

$K = 24$
$a = 1$
$b = 2$
$n = ?$

$$|E_S| \le \frac{K(b-a)^5}{180n^4} \le 0.0001$$

$$\frac{24(2-1)^5}{180n^4} \le 0.0001$$

$$\frac{24}{180(0.0001)} \le n^4$$

$$\sqrt[4]{\frac{24}{180(0.0001)}} \le n$$

$$6.04 \le n$$

$$n = 8$$ ← n must be even.

$$= \int \left(\frac{-1}{u+1} + \frac{2}{u+2} \right) du$$

$$= -\ln|u+1| + 2\ln|u+2| + C$$

$$= \boxed{-\ln|e^x+1| + 2\ln|e^x+2| + C}$$

$$= -\ln|e^x+1| + \ln|e^x+2|^2 + C$$

$$= \ln \frac{(e^x+2)^2}{(e^x+1)} + C$$

Laws of Logs

1. $\ln(AB) = \ln A + \ln B$
2. $\ln(A/B) = \ln A - \ln B$
3. $\ln(A^t) = t \ln A$

§7.7 # 20

a) Find the approximations T_{10} and M_{10} for $\int_1^2 e^{1/x}\,dx$

SOLUTION

$a = 1$

$b = 2$

$n = 10$

$\Delta x = \frac{b-a}{n} = \frac{2-1}{10} = \frac{1}{10} = .1$

$x_0 = 1$ $\quad +\Delta x$

$x_1 = 1.1$ $\quad +\Delta x$

$x_2 = 1.2$

$x_3 = 1.3$

$\vdots$

$x_9 = 1.9$

$x_{10} = 2.0$

Midpoints

$\bar{x}_1 = \frac{x_0 + x_1}{2} = \frac{1+1.1}{2}$

$= \frac{2.1}{2}$

$= 1.05$

$\bar{x}_2 = \bar{x}_1 + \Delta x$

$= 1.15$

$x_3 = 1.25$

$x_4 = 1.35$

$x_5 = 1.45$

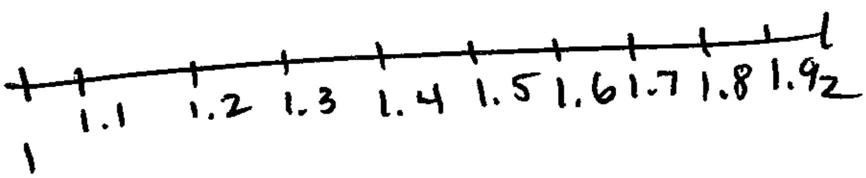
$x_6 = 1.55$

$x_7 = 1.65$

$x_8 = 1.75$

$x_9 = 1.85$

$x_{10} = 1.95$



Midpoint Rule

$$M_n = \Delta x \left[f(\bar{x}_1) + f(\bar{x}_2) + \cdots + f(\bar{x}_n) \right)$$

$$f(x) = e^{1/x}$$

$$= .1 \left[e^{1/(1.05)} + e^{1/1.15} + e^{1/1.25} + e^{1/1.35} + e^{1/1.45} + e^{1/1.55} + e^{1/1.65} + e^{1/1.75} + e^{1/1.85} + e^{1/1.95} \right]$$